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Calculation of monthly mean temperature by Köppen's formula in the Norwegian station network



Øyvind Nordli and Ole Einar Tveito



**”Es ist zu bedauern, dass dieses Missverständnis die k-Methode in
Misskredit gebracht hat, so dass sie bis jetzt ausser in Norwegen keine
Verwendung gefunden hat“.**

B.J. Birkeland 1935

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<p>Abstract</p> <p>As probably the only institute in the world the Norwegian Meteorological institute is using Köppen's formula for the calculation of monthly mean temperature at manual stations. At the end of 2004 there were 66 automatic stations with sufficiently long data series available for calculation of Köppen's constant. So far the constant was only known from spatial interpolation on maps based on a few thermograph stations. Using the whole material of 66 stations the constant was found to have a small scale variability making mapping of the constant insufficiently accurate for practical use for the whole Norwegian area. The constant was, however, closely related to the Daily Temperature Range (DTR) that is a known variable for the entire historical network of stations. During winter the constant was closely related to the latitude. Thus, in stead of mapping the constant, regression equations were established with the latitude (months November – March) and DTR (months April – October) as input parameters.</p> <p>Without minimum temperature Köppen's formula can not be use, i.e. prior to 1876 for the Norwegian station network. Testing the classical c-formula and Føyn's formula, the latter was chosen for practical use. Føyn's formula was less robust for errors during the winter months than the classical formula, but gave smaller errors during the other seasons.</p>	
<p>Keywords Köppen's formula, k-formula, mean temperature, Føyn's formula, DTR</p>	

Disciplinary signature	Responsible signature
	
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1 Introduction

At the Norwegian Meteorological Institute automatic stations are currently replacing manual stations. In the future they are thought to be the only ones in use. They are logging temperature every hour making the calculation of daily as well as monthly means straight forward. Daily and monthly means are by definition the arithmetic means of the hourly observations.

One could think that the problem of calculating true monthly means based on observations at a few fixed hours is not any longer a problem of particular interest, but this is not the case for the study of long-term historical series. Homogenous series are a necessity for such studies. This raises the question how to link the manual series to the automatic ones. The answer is that the automatic stations give affluent information of the daily temperature wave that may be used to test earlier formulae for monthly mean calculations.

2 The historical perspective

A new formula for calculation of mean monthly temperature was introduced by Köppen (1888), and in 1890 the formula was taken into use by Deutsche Seewarte for the coastal Baltic Sea stations (Grossmann 1892) from Borkum (west) to Memel (east). In his publication Grossmann concluded, however, that further estimation of the k-values is necessary to be sure that the new formula is better than the old one. Leyst (1892) also analysed the formula of Köppen and concluded that the formula did not lead to any improvement, whereas Birkeland (1935) criticised Leyst for not having performed a relevant investigation, and for having put forward misleading conclusions. Birkeland states that no other country than Norway (1935) uses Köppen's formula. He thinks that the formula came into discredit by Leyst's article.

In Norway different formulae for monthly mean temperature (T_m) were used during the period 1876 to 1889, but from 1890 Köppen's formula was adopted as standard for the Norwegian network. Also recalculation of old data were performed back to 1876, but going further back in time was not possible due to the lack of daily minimum temperature. Thus, from 1876 to present the same formula has been in use for the Norwegian station network, a period of about 130 years. The formula is commonly written:

$$T_m = T_f - k(T_f - T_n) \quad (1)$$

where T_f is the mean of the three observations at fixed hours (morning, midday, and evening), T_n is the daily minimum temperature, and k is the so called Köppen's constant.

The magnitude of k depends on the location, month, and the time of observation during the day. Often the constant k is called the temperature factor or the k-value. The k-values were firstly calculated from hourly observations in Oslo, Bergen, Trondheim, Alta, Vardø, and Spitsbergen. For the other stations the k-values was established by map interpolations. It is sufficient to present k with two digits. Its range is from zero to 0.30.

During the long-lasting use of the formula the Norwegian network has undergone several changes both in the observation times and the definition of the temperature day. These have acquired new calculations of Köppen's constants, altogether 6 different sets of constants have been necessary in order to cope with all changes (table 1). In the present article the sets are

given numbers from 1 to 6 (k_1 to k_6) starting with the present values going successively backward in time. There have been no changes since July 1949.

Table 1. Different k-values defined according to observation times (Hours), temperature day (Day), and setting and reading of the minimum thermometer. In the table the hours are given in UTC. However, in the period 1876 – 1920 some stations observed according to local time, and not exactly to the time shown in the table. Some stations (in particular at telegraph stations) observed according to Christiania Time, which means 18 min. later than shown in the table.

k	Period	Time	Hours	Day	Period of t_n
k_1	1949.07 – present	UTC	06, 12, 18	18 – 18	18 – 18
k_2	1949.01 – 1949.06	UTC	07, 12, 18	18 – 18	18 – 18
k_3	1938.01 – 1948.12	UTC	07, 13, 18	18 – 18	18 – 18
k_4	1920.07 – 1937.12	UTC	07, 13, 18	07 – 07	07 – 07
k_5	1894.01 – 1920.06	UTC	07, 13, 19	07 – 07	07 – 07
k_6	1876.01 – 1893.12	UTC*	07, 13, 19	07 – 07	19 – 07

3 Dataset and method

Temperature is measured every hour at the Norwegian automatic stations enabling calculation of monthly mean temperature according to the international definition. This is a much larger material that could be provided by the old thermographs. Among the automatic stations are also mountain stations and lighthouse stations. Automation of the lighthouses has been put forward by the authorities in order to save expensive labour at often remote and hardly accessible islands. Many of the lighthouses also perform meteorological observations that were automated too. Thus, the lighthouse stations are better represented in the data set available for k-value calculation than should be expected according to their number in the entire station network. Altogether 13 of 66 available stations in the data set are lighthouse stations.

For the automatic stations T_m is known and by reformulating (1), k may be written:

$$k = \frac{T_f - T_m}{T_f - T_n} \quad (2)$$

The k-values were plotted on maps to get an idea of their geographical variation, but no clear pattern was easily recognised. However, two features were evident. During winter the k-values have a north-south gradient and during summer the lighthouse stations were seen to have lower k-values than the neighbouring stations. The other variations were thought to be caused by local climates that could not easily be caught by a dataset of few stations spread over such a large area. One variable that could be closely connected to the k-values was thought to be the Daily Temperature Range (DTR) defined as $T_x - T_n$, where T_x is the mean monthly daily maximum and T_n is the mean monthly daily minimum.

The monthly k_1 values were plotted against DTR as well as latitude like in the example for July (figure 1). Only a subset of the data set was used, namely older stations with DTR easily available. One outlier was detected, not only for July, but also for several other months. This series was removed from the data set before further processing of the data. Leaving out also two arctic stations, the subset contained 39 series. The k-values for the lighthouse stations are

lower than for the rest of the data set, but they seem to be nicely distributed around the regression line.

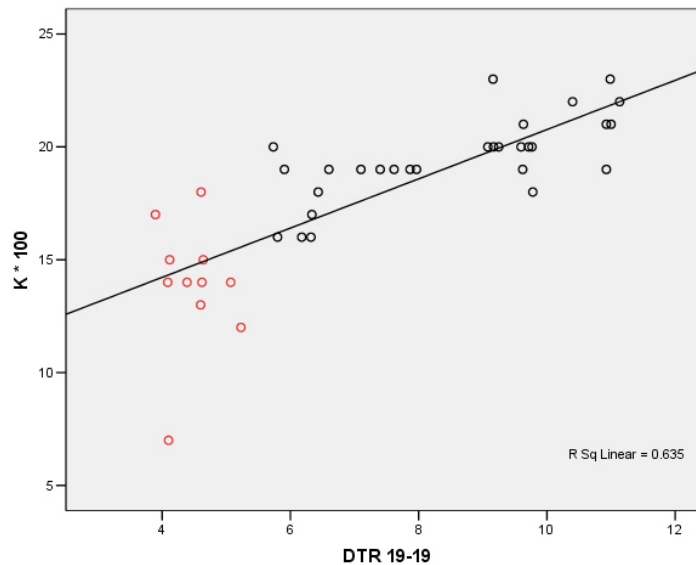


Figure 1. The relationship between k_1 -values ($\cdot 100$) and the DTR ($^{\circ}\text{C}$) in July. The lighthouse stations are marked by red rings. The station 59110 Kråkenes fyr in the lower left of the July diagram is regarded as an outlier.

Latitude and DTR was found to be correlated for all months. The correlations are weaker during winter and highest during summer and autumn, e.g. September ($R = -0.50$) and January ($R = -0.28$). For all months but January the correlation was significant. Due to this correlation there is a risk for over fitting the regression equation. In order to keep this under control stepwise multiple regression analysis was chosen with latitude and DTR as predictors. The number two predictor was only adopted if its significance was 5 % or better according to an F-test.

For all months one of the variables was thrown out for not passing the selection criterion so that every regression equation does not contain more than one predictor. The analyses were only performed for k_1 and k_3 . The results obtained for the two sets of k-values were much similar.

4 Results

4.1 The Köppen period 1876 – present

For the season from November to March latitude was the chosen predictor, whereas in the season April to October DTR was chosen. This may be due to the different nature of DTR during those seasons. During winter the largest proportion of the DTR variations is aperiodic, caused mainly by the shifts of moving low and high pressure systems, whereas in the rest of the year DTR variations are mainly periodic, due to very little global radiation during night and much more intense global radiation during day. The regression correlation between DTR and the k-values are approximately 0.8 during the summer months (table 2).

Table 2. Köppen's constant $\cdot 100$ (k_1 and k_3) estimated from stepwise regression analysis with latitude (Lat) and Daily Temperature Range (DTR) as predictors. a_0 and a_1 are regression constants, R is the regression correlation, and s the standard deviation of the residuals (cross-validated)

$k_1 \cdot 100$ (1949.07 – present)					$k_3 \cdot 100$ (1938.01 – 1948.12)				
Month	a_0	a_1	R	s	Month	a_0	a_1	R	s
1	49.50	-0.737*Lat	0.84	1.8	1	63.90	-0.946*Lat	0.85	2.2
2	30.98	-0.414*Lat	0.59	2.7	2	47.15	-0.648*Lat	0.70	2.7
3	25.99	-0.306*Lat	0.50	2.5	3	54.29	-0.680*Lat	0.68	3.3
4	8.35	0.737*DTR	0.60	2.1	4	10.52	1.370*DTR	0.78	2.4
5	12.30	0.803*DTR	0.73	1.9	5	16.11	1.011*DTR	0.82	1.7
6	12.26	0.805*DTR	0.82	1.5	6	16.56	0.893*DTR	0.79	1.8
7	10.91	0.975*DTR	0.82	1.6	7	14.16	1.236*DTR	0.77	1.9
8	7.28	1.078*DTR	0.84	1.8	8	11.62	1.365*DTR	0.87	1.9
9	2.81	0.836*DTR	0.74	1.5	9	9.94	1.667*DTR	0.88	1.6
10	-2.43	1.461*DTR	0.69	2.1	10	-1.29	1.810*DTR	0.64	2.9
11	26.72	-0.364*Lat	0.54	2.1	11	35.79	-0.503*Lat	0.76	1.7
12	33.64	-0.515*Lat	0.76	2.0	12	37.47	-0.575*Lat	0.76	2.0

In calculating the standard deviation of the residuals, a technique called leave-one-out cross-validation was used. The residuals were treated case by case, and different regression equations were used each time. When a residual for a case was derived, that case was deleted from the data, and the regression was based on the remaining N-1 cases. This procedure was repeated for each residual in turn. Thus, the case that was subject for validation had no influence on the regression used for calculation of that residual. From the table is seen that s varies from about 0.015 (summer and autumn) to about 0.025 during winter and early spring.

Changes in the observation times have not been larger than about one hour. Therefore there must be a strong relationship between the different sets of k-values. This was tested by choosing the currently used k-value, k_1 , as the only predictor in regression analyses with the historical k-values (k_2, k_3, \dots, k_6) as predictands. The regression equations are written:

$$k_i = b_i \cdot k_1 \quad (3)$$

If the k_1 value is zero, also the historical k-values for that station and month should be zero, and regression through the origin (the no-intercept model) was used. Thus, R^2 measures the proportion of the variability in the dependent variable about the origin accounted for by the regression. For all regressions the R was very high, varying from 0.97 to 1.00. The standard deviation of the residuals was about 0.01.

The regression was performed with the subset of 39 stations as well as the whole data set (except one outlier station and two Arctic stations). The result differed very little between the subset and the whole data sets (table 3), and for further work the results based on the whole data set was adopted. During the darkest months of the year, November – February, the b-values in equation (3) are noisy, but the k-values are small for those months so that for practical purposes also the historical k-values may be assessed with sufficient accuracy. For the other months the regression coefficients (b-values) and thus the historical k-values are qualitatively interpreted as follows:

The b_2 -values (observations at 07, 12, and 18 UTC) are larger than $b_1 = 1$ (06, 12, and 18 UTC) due to later morning observation (Table 1), and b_3 -values (07, 13, and 18 UTC) are larger than b_2 -values due to later midday observation (Table 3). The b_4 -values are somewhat

smaller than the b_3 -values due to changed temperature day, but this effect is so small that it may be neglected for summer and winter, but should be taken into account for autumn and in particular for spring. These findings are in line with the results from an investigation using other methods (Nordli 1997; Tuomenvirta et al. 2000). The b_5 -values (07, 13, and 19 UTC) are lower than the b_4 -values due to later evening observation. The observation times connected to b_5 are just a displacement of 1 hour compared to the present standard. Cooling in the evening is slightly more than compensated by later midday and in particular later morning observations (this is why b_5 is close to 1). Finally the b_6 -values are lower than the b_5 -values, which may be explained by the different period of the minimum temperature. The b_6 -values are based on nightly minima, whereas the b_5 -values are based on daily (day and night) minima. During summer, however, there is almost no difference between the two sets of constants as minimum very seldom occurs at daytime.

Table 3. Regression coefficients for historical k-values using the present k-value (k_1) as predictor in a regression through origin analysis. The b_2 coefficient is the regression coefficient for the k_2 -value or: $k_2 = b_2 \cdot k_1$, $k_3 = b_3 \cdot k_1, \dots, k_6 = b_6 \cdot k_1$. The coefficients marked with ' are based on the subset of 39 series, whereas unmarked coefficients are based on the data set of 66 series.

Month	b2'	b2	b3'	b3	b4'	b4	b5'	b5	b6'	b6
1	1.00	0.99	1.25	1.19	1.14	1.10	1.02	0.98	1.44	1.38
2	0.97	1.00	1.29	1.17	1.11	1.05	0.87	0.84	1.16	1.22
3	1.43	1.42	1.67	1.58	1.45	1.41	1.09	1.09	1.32	1.29
4	1.40	1.41	1.48	1.48	1.41	1.42	1.06	1.10	1.15	1.16
5	1.25	1.24	1.29	1.28	1.29	1.28	1.07	1.07	1.10	1.09
6	1.22	1.22	1.27	1.26	1.26	1.26	1.08	1.08	1.11	1.10
7	1.23	1.23	1.29	1.29	1.28	1.28	1.08	1.09	1.11	1.11
8	1.34	1.33	1.41	1.41	1.41	1.40	1.07	1.08	1.12	1.12
9	1.70	1.64	1.88	1.77	1.78	1.71	1.31	1.27	1.46	1.40
10	1.28	1.26	1.48	1.40	1.33	1.29	1.02	0.99	1.31	1.26
11	0.98	0.97	1.03	0.93	0.96	0.88	0.76	0.71	1.12	1.05
12	1.01	0.95	1.00	0.87	1.00	0.87	0.85	0.77	1.33	1.17

4.2 The pre-Köppen period, 1864 – 1875

Before 1876 minimum thermometer was not standard in the Norwegian network. The only thermometer readings were at the observing times 8, 14 and 20 local times (table 1). Realising that the monthly mean of the morning and evening observations gave results close to the true monthly mean, these observations were used for monthly mean temperature calculation, whereas the midday observation was omitted. The formula (hereafter called the classical c-formula) may be written:

$$T_m = T_g + c \quad (4)$$

where c is a correction term that has to be calculated and T_g is the mean of the morning and evening observations. The c -value can be calculated by (4b) if T_m is known.

$$c = T_m - T_g \quad (4b)$$

By combining equations (1) and (4) the constant c is given by

$$c = (T_f - T_g) - k(T_f - T_n) \quad (5)$$

If the k-value is zero, it means that temperature does not vary systematically during the day, i.e. $T_f = T_g$, and $c = 0$ according to (5).

Later Føyen (Birkeland 1935) introduced a Köppen-like formula

$$T_m = T_g + k_g \cdot (T_2 - T_g) \quad (6)$$

where T_2 is the midday observation and k_g is a constant that has to be calculated. Hereafter this will be referred to as Føyen's formula. Equation (6) may be reformulated

$$k_g = \frac{T_m - T_g}{T_2 - T_g} \quad (7)$$

For the summer months the mean values of T_m is lower than T_g leading to negative k_g , i.e. the higher midday observation the lower T_m . This rather curious situation is no paradox. It can be explained by extensive drops in temperature during night in clear sky situations not fully compensated for by high T_2 .

An expression for k_g may also be found by combining equations (1) and (6).

$$k_g = \frac{1}{3} - \frac{T_f - T_n}{T_2 - T_g} \cdot k \quad (8)$$

If k and T_n are known, k_g may be calculated by (8).

By using the formula on the present data set it turns out that k_g might take unrealistic values for stations in northern Norway and for some of the lighthouse stations, in both cases during the season November - January. The denominator might take values close to zero leading to unrealistic k_g -values very far from zero. Used on an independent data set this might cause huge errors. If there is nothing to gain using formula (6), the simpler and more robust formula (4) should be used.

The alternative formulae (4) and (6) were tested for 8 selected series against true monthly means. In table 4 the results are given as the standard deviation of the twelve monthly differences for each of the stations. Using Føyen's formula a large standard deviation occurs for the northernmost station Slettnes fyr but the reason is an outlier for December. For the rest of the series there are only small differences in the standard deviations between the series when all months are considered. For some individual months Føyen's formula gives far better results (lower s) than the classical formula.

Table 4. Standard deviation of the differences between true monthly means and monthly means calculated by the classical formula (4) and Føyn's formula (6) for 8 selected stations: 00180 Strømtangen fyr, 16610 Fokstugu, 18700 Oslo – Blindern, 39040 Kjevik, 52530 Hellisøy fyr, 71000 Steinkjer, 90400 Tromsø – Langnes, 96400 Slettnes fyr

Stations	00180	16610	18700	39040	52530	71000	90400	96400
s (Føyn)	0.12	0.14	0.13	0.16	0.07	0.14	0.10	0.43
s (c-values)	0.16	0.15	0.16	0.21	0.09	0.18	0.10	0.09

5 Practical calculation of new k-values

For new stations the calculation of k-values has no practical interest as all newly established stations are automatic and observe temperature every hour. Köppen's method is replaced by an arithmetic mean calculation of the hourly values. For old stations, however, automations will lead to a shift of formula. A wrong k-value will therefore influence the homogeneity of the series, and thereby the temperature trend, whereas before the automation it might influence the absolute values only. Thus, for historical stations wrong k-values may be critical for the homogeneity of their series.

For the historical stations the DTRs are known and can be used directly as input variables in the regression equation. The last term in Köppen's formula (1) is the $k \cdot (T_f - T_n)$. From figure 2 is seen that the difference in the parenthesis amounts to about 4 °C most of the year with somewhat lower values during autumn and early winter. Setting $T_f - T_n = 4$ °C and $s = 0.025$, the standard error of the estimated monthly mean temperature amounts to 0.1 °C. In the data set there is also some continental series with larger $T_f - T_n$. In exceptional cases $T_f - T_n = 8$ °C does occur leading to a standard error of 0.2 °C. For most purposes even a standard error of 0.2 °C is acceptable as local conditions at the measuring site may influence measurements even more. For the study of trends in a series, however, a standard error of 0.2 °C may hamper the analysis and increase the risk for assessing false trends. This risk is reduced by also using the k_1 value for the calculation of the historical k-values. A somewhat wrong estimation of the k_1 values also affects the other k-values in the same direction. This is, however, wanted as it reduces the risk for inhomogenous series.

For practical calculation of the k-values, k_1 should be calculated by linear regression using the coefficients in table 2,

$$k_1 = a_0 + a_1 \cdot DTR \quad (\text{April} - \text{October}) \quad (9)$$

$$k_1 = a_0 + a_1 \cdot Lat \quad (\text{November} - \text{March}) \quad (10)$$

Historical k-values should be calculated by equation (3), where b is taken from table 5. The coefficients in table 5 are the same as in table 3 except for the noisy coefficients during winter. In winter the historical k-values should be approximately equal to the k_1 -values, which are obtained by setting the coefficient equal to 1.

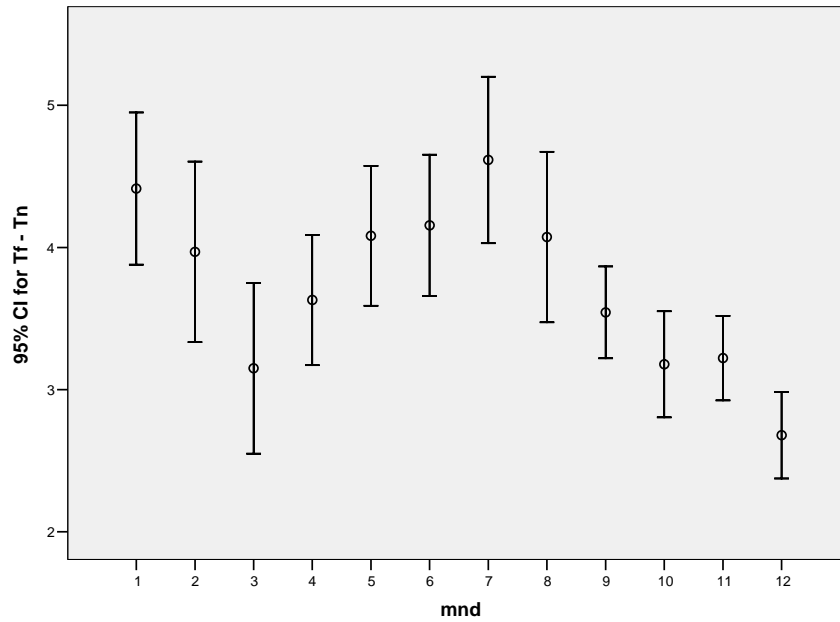


Figure 2. Difference between the mean value of the temperature at the three fixed hours (06, 12, and 18 UTC) and the daily minimum temperature. The 95 % confidence interval is also shown.

Table 5. Coefficients for historical k-values using the present k-value (k_1) as input variable. The b_2 coefficient is the regression coefficient for the k_2 -value or $k_2 = b_2 \cdot k_1$, $k_3 = b_3 \cdot k_1$ $k_6 = b_6 \cdot k_1$.

Month	b_2	b_3	b_4	b_5	b_6
1	1.00	1.00	1.00	1.00	1.38
2	1.10	1.17	1.05	1.00	1.22
3	1.42	1.58	1.41	1.09	1.29
4	1.41	1.48	1.42	1.10	1.16
5	1.24	1.28	1.28	1.07	1.09
6	1.22	1.26	1.26	1.08	1.10
7	1.23	1.29	1.28	1.09	1.11
8	1.33	1.41	1.40	1.08	1.12
9	1.64	1.77	1.71	1.27	1.40
10	1.26	1.40	1.29	1.00	1.26
11	1.00	1.00	1.00	1.00	1.05
12	1.00	1.00	1.00	1.00	1.17

The monthly mean temperature before 1876 should be calculated from Føyn's formula or from the c-formula, taking the c and the k_g values from formulas (5) and (8) respectively. It is further suggested that the input parameters should be taken from the period 1876 – 1893 involving $k_6 = b_6 \cdot k_1$. Føyn's formula is not robust used for the months November, December, and January. Therefore the c-formula is recommended for those months. For the rest of the year Føyn's formula tends to give more accurate estimates for the monthly means. Therefore Føyn's formula is recommended for the months from February to October. In occasions where the k-value is zero also the c-value and Føyn's constant should be set to zero.

6 Comparison with earlier sets of Köppen constants

Some of the automatic stations have a long history as traditional manual weather stations, where old k-values based on thermograph data were in use. Comparison with the new data shows very little difference during most of the year except for some spring and autumn

months. The most striking example is September where the difference amounts to 0.04. The coefficient b_3 (table 3) shows high values for spring and autumn months, in particular for September. At those seasons the time of the morning observation is near to the sun rise with large impact on the morning temperature. At present the morning observation is taken at 07 CET that might be too early for heating of the air, whereas this might have changed one hour later. This might not have been fully reflected in the old k -values.

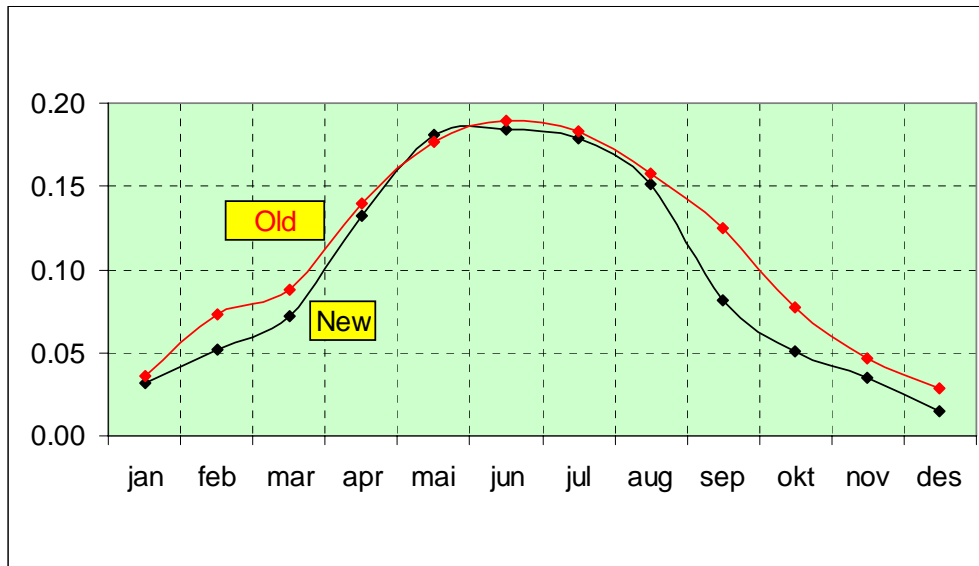


Figure 3 Comparison between old and new k_1 values

7 Conclusions

Köppen's constant was found to have a small scale spatial variability making mapping of the constant difficult. The constant was however, closely connected to the Daily Temperature Range (DTR).

Føyn's formula for the period before the Norwegian stations were equipped with minimum thermometer (prior to 1876) is less robust for the winter months than the classical c -formula, but gives smaller errors during the other seasons.

8 Recommendations for calculation of monthly means

- Köppens constant (present k -value) for three daily observations (06, 12, 18 UTC) should be calculated from the Daily Temperature Range (DTR)
- Historical k -values should be calculated by using the present k -values
- Before the minimum thermometer was introduced in the network (prior to 1876), the c -formula should be used for the months November, December, and January (alternatively Føyn's formula may be used with $k_g = 0$), whereas Føyn's formula should be used for the rest of the year.
- The k -values calculated by the DTR should be replaced by k -values calculated directly from the hourly observations at each station when long series (presumably more than 20 years) of hourly observations are available.

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