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Waves in sea ice

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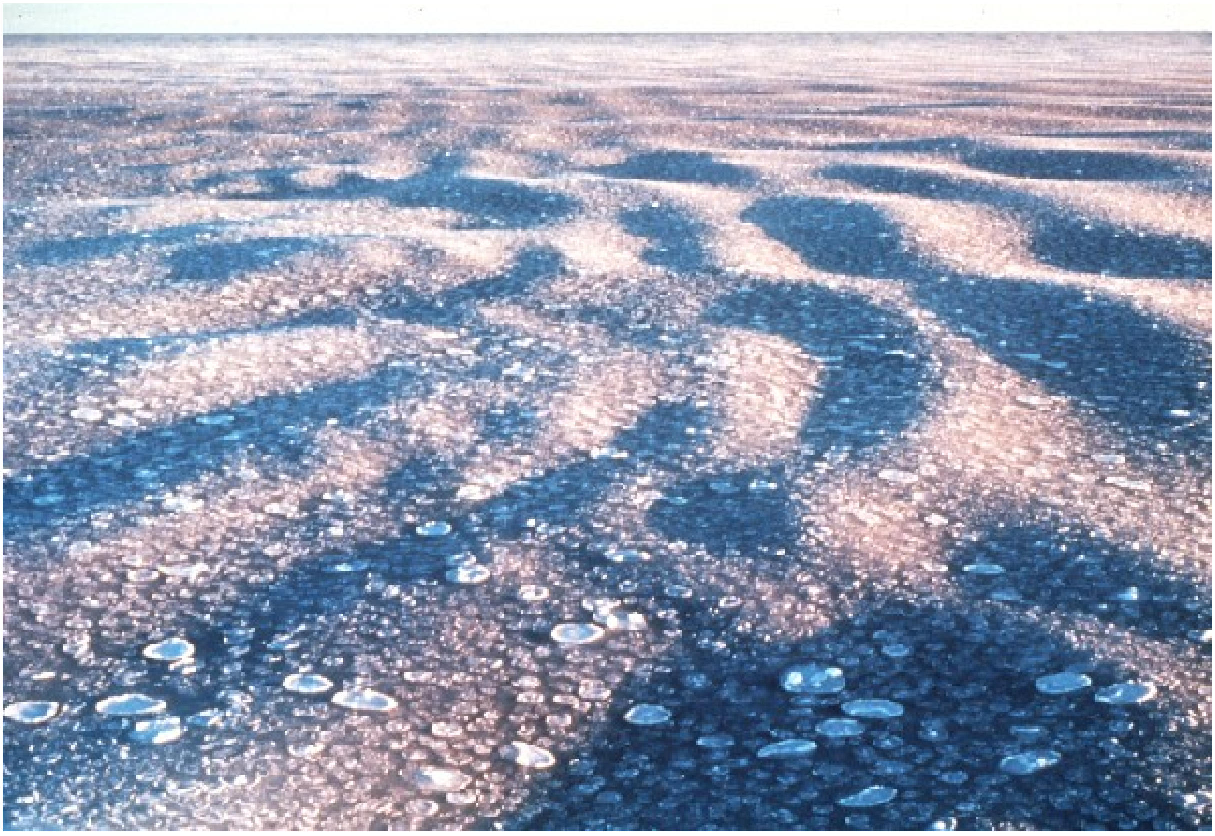

Wave in pancake ice

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Abstract

This study provides an overview on how waves and ice interacts in the ocean. The most relevant ice characteristics found in the ocean is described, and their role on the wave dynamics is outlined. The impacts of the waves on the ice field are also described to some extent. Two different models for wave ice interaction, namely the "standard" mathematical model describing wave scatter due to ice floes, and a model describing waves in slush ice are presented in some detail. Energy transport models applying a Boltzman type of scattering are also briefly described.

It is concluded that wave ice interaction is challenging to describe in detail, and mathematical models describing wave ice interaction are often very complicated mathematically and employ advanced mathematical methods. Nevertheless, there are some simple fundamental principles that can be applied for modeling the wave energy in ice conditions. It is concluded that met.no has the capacity to implement a wave-in-ice model although it may require a considerable effort.

The authors suggest that there is a mismatch between the theoretical developments describing wave ice interactions, and observational and laboratory studies. Interlinked studies using dedicated field experiments, laboratory experiments, and numerical models targeting realistic physical system are needed for advancing the present state-of-the-art knowledge of wave ice interaction.

Keywords
Wave, ice, model

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Executive summary

This study provides an overview on how waves and ice interacts in the ocean. The most relevant ice characteristics found in the ocean is described, and their role on the wave dynamics is outlined. The impacts of the waves on the ice field are also described to some extent. Two different models for wave ice interaction, namely the "standard" mathematical model describing wave scatter due to ice floes, and a model describing waves in slush ice are presented in some detail. Energy transport models applying a Boltzman type of scattering are also briefly described.

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Contents

1 Introduction

StatoilHydro runs several operations in areas that is covered with ice or having a risk of being affected by ice. For safe operations in these areas it is advantageous to have precise knowledge of wave and ice conditions. Today there is no comprehensive forecasting system that deals with the wave-ice system albeit the crucial role of the wave-ice climate for planning operations in ice-covered areas. To provide further information on the possibility for launching an operational wave-in-ice service, StatoilHydro has asked the Norwegian Meteorological Institute (met.no) to provide a state-of-the-art description of the present knowledge of wave ice interaction and the modeling of waves in ice, and the result of this literature study is presented in this report.

The coupling between waves and ice involve fluid mechanics and its coupling to floating objects, which also have certain flexibility, and is extremely complex. The complexity of the physical system implies that the mathematical models must be equally complex. Many of the basic physically realistic scenarios cannot be solved even using the most advanced mathematical and numerical methods as of today. There exist a wide literature on wave and floating object interaction applicable to wave ice interaction. However, many of these studies are mathematically complex, and it may be difficult to apply the idealizations needed for a geophysical usable model. Furthermore, theoretical frameworks describing wave transition through non-homogenous media (such as electromagnetic radiation through various media) can be used for describing wave transition through an ice field. Again it may be stated that these models are complex and are not readily applicable into a model describing waves in a geophysical context.

The ice field has a clear impact on the wave field, but the wave field also has a strong impact on the ice field. The mutual interactions between the wave field and the ice characteristics add further to the complexity of the system. The ice field respond quickly to an incoming wave field, the ice is cracked into smaller floes creating an entirely new ice field within hours. In addition to the mutual interactions between the wave and ice fields, the wave and ice fields have a distinct influence on ocean currents and the atmospheric flow adding to the complexity of the geophysical system involving the wave-in-ice characteristics.

It is concluded that wave ice interaction is very complex and mathematical theories describing the system are complex. However, certain properties of the system suggest that it can be predicted using fairly simple models; for instance, the energy decreases exponentially from the ice edge albeit the length scale of the decline varies with ice characteristics. However, the ice field is a strong function of the wave field, and the wave field is a strong function of the ice field. If this implies that the final model will be more reliable or not is still an open question but the possibility exists.

One of the obstacles in developing a forecasting system for describing waves-in-ice is that the model needs to be validated. Today, there are only a few studies that can be used to validate a wave-in-ice model. Further observations may be needed. Another difficulty with accurate predictions of the coupled wave-ice system is the small scales of the system. The wave-ice zone has scales of order 1-50 km and it changes rapidly with changing forcing conditions. Such small scales in combination with high variability will certainly push any geophysical model to its limits.

It is concluded that met.no has the basic operational models (i.e., atmospheric, ocean, ice and wave models) needed for a wave-in-ice forecasting system. However, the wave model and the ice model will need substantial revisions to accommodate the wave-in-ice dynamics. Furthermore, some theoretical development must also be considered. The models will also require higher resolutions to be able to make accurate prediction of the wave-in-ice system.

During this literature study it has become clear that there are at least three different aspects of the coupled wave-ice system that is relevant for predicting the wave and ice climate in a region.

- The wave amplitude that is affected by ice coverage and ice thickness.
- Ice dynamics and ice coverage, which to a large extent is determined by the wave forcing.
- The possibility to use observations on wave parameters, e.g., by SAR radar or satellites, to estimate the ice thickness [*Nagurny, et al.*, 1994; *Wadhams, et al.*, 2002; *Wadhams, et al.*, 2004].

The first two items are indeed intimately coupled; however, in most studies there is no direct coupling such that there are no holistic studies of the wave-ice system as of today[1]. The third aspect is not directly a part of the study presented here: however, it is an important application that can shed some light on the ice thickness, which is an important parameter for the wave-ice coupled system

This report focuses on the more practically oriented description of waves in ice-covered areas and therefore we neglect a number of research articles that are considered to focus more on the mathematical details of the ice-wave interaction than on the more practical issues of the problem. Here some choices have to be made and the authors certainly have neglected studies that are important for the subject but nevertheless have been overlooked due to the ignorance from the present authors side. More detailed reviews can be found elsewhere [*Lavrenov*, 2003; *Squire*, 2007; *Squire, et al.*, 1995; *Wadhams*, 2000].

Some basic physics on waves in ice

For waves in the open water gravity is the dominant restoring force and hence the name gravity waves. In an ice covered sea the stiffness of the ice will contribute to the returning force of the sea surface and the waves under stiff sea ice are called flexural-gravity waves. It should be acknowledged that variable ice-thickness, and the corresponding variability in flexural force from the ice cover, will affect the wave speed and thus the direction of propagation[2]. Furthermore, waves for a given frequency tend to be longer under sea ice than in open water conditions, and their group velocity becomes smaller such that the wave amplitude increases (see Appendix A). Besides the direct influence of the ice cover on the restoring force of the water-ice surface, in-homogeneities in the ice cover, such as ridges, cracks in the ice, and open water, will give rise a certain scatter of the waves, which will influence the wave height (see Appendix B). Over the last few years there have been a large number of theoretical studies on these scattering processes; however, most studies are more devoted to the mathematical description of the scattering process rather than towards a real

[1] This is of course not entirely true but most studies tend to focus on either of the two aspects.

[2] Snells law predicts that waves that experience a change in propagation speed will bend with respect to the normal; there will also be some scattering of the wave at a sudden change of the phase speed.

life applications. Accordingly, many of these studies are not easy to access for scientist outside the field of applied mathematics.

Another track in describing waves under the influence of ice takes the limit that the ice is represented by very small particles such that the ice is fully diluted into a water-ice suspension. The water-ice suspension (or slush) is very viscous due to the interaction of the ice particles diluted in the water; furthermore, the water-ice slush is very buoyant such that it floats and forms a relatively homogenous upper layer. The high viscosity of the water-ice slush implies that waves are rapidly damped by viscous affects; here scattering of waves are not considered. One framework describing this system is described in Appendix C.

Summarizing, in the literature two types of ice-wave theories are encountered:

- The presence of flexible or solid ice floes.
- The case when ice is treated as ice slush, such that the ice can be treated as viscous fluid layer.

In natural conditions, either of the two types of ice is seldom observed and it appears that most ice conditions have some mixture of flexible ice floes and small slush ice. Today, there are no studies where both these features are described in a common way.

The main summary of the wave propagation in ice covered areas is given in Section 2. Sec. 2.1 describes field studies of both ice properties and wave-in-ice dynamics. Laboratory experiments are covered in 2.2, while theoretical studies are summarized in Sec 2.3. Section 2.3.1 outlines the ”standard” mathematical description used to model scatter from ice floes. The viscous slush ice model is described in Sec. 2.3.2, while energy transport models employing wave scatter from ice floes is described in Sec. 2.3.3. Section 3 is devoted to geophysical models. 3.1 outlines the spectral wave models that are used in geophysical context, a brief presentation of a wave model where wave ice interaction is included is given in 3.1.1. The most relevant operational model at met.no is outlined in Sec. 3.2. Section 4 provides an overview of processes that must be included into a forecasting system, and the possibility to include wave-in-ice characteristics into the met.no operational system is discussed in 4.2. Section 4.3 gives some suggestions for future studies. Appendix A outlines some general wave characteristics while Appendix B gives a more thorough outline of the “standard” model for describing wave scattering. Appendix C discusses the basic outline of a viscous model for slush ice.

2 Summary of literature regarding wave propagation into sea ice

In this section we will categorize different works as field studies, laboratory studies, or theoretical studies. Of course, not all works can be categorized in such way and different studies may appear in more than one section. Most of what we know on ice field characteristics, and its response to wave action, comes from field experiments, which are describe in section 2.1. This section also gives short presentations of the most pertain ice types that are relevant for waves-in-ice. Some relevant laboratory experiments are described in section 2.2; section 2.3 is devoted to theoretical description of wave-in-ice dynamics. In section 2.4 the possibility to use observations on waves to estimate the ice thickness is briefly described; the section also contains a discussion on energetic waves that may appear in solid ice.

2.1 Field studies

Field studies represent the most straightforward way to obtain new understanding of the wave-ice interaction. However, the problems in obtaining a useful dataset should not be underestimated; this is clear from the relatively few useful observations that have been made. The most comprehensive measurements were made by Scott Polar Institute at University of Cambridge in the mid seventies [*Squire*, 1984; 1995; *Squire, et al.*, 1995; *Squire and Moore*, 1980; *Wadhams*, 1973; 2000; *Wadhams, et al.*, 1987; *Wadhams, et al.*, 1986; *Wadhams, et al.*, 1988; *Wadhams, et al.*, 2006]; albeit there have been new observations since these early measurements they have not advanced our knowledge in any dramatic way. However, we start by describing ice types that are important for this study.

2.1.1 Various ice types discussed in this study

When considering ice there are several types of ice and here we consider a short outline of some ice characteristics that are relevant for this study. It should be recognized that when ice forms in the ocean small pockets of sea water will become trapped in the ice. Thus, as salt water has a lower freezing point these small pockets will not freeze altogether. The physical/mechanical behavior of sea ice may thus behave somewhat different as compared to ice created from freshwater.

2.1.1.1 Grease ice

As the water cools and starts to reach the freezing temperature small ice crystals are formed, mainly in the form of small discs with size of order 2-3 mm [*Martin and Kauffman*, 1981]. In calm conditions these floats on the surface, while a more turbulent or wavy environment (which is the normal state of the upper ocean) implies that these crystals are suspended deeper into the sea. The discs tend to stuck to each other to minimize their thermodynamics energy and water with high concentrations of small ice crystals is therefore very viscous [*Martin and Kauffman*, 1981]. This type of ice is usually called grease ice or frazil ice and the thickness of the ice slush grease may exceed 1 meter under right conditions [*Martin and Kauffman*, 1981; *Wadhams*, 2000].

2.1.1.2 Pancake ice

When freezing continues the grease ice concentration increases and it finally reaches a transition point where ice crystals start to form small cakes. The size of the cakes depends on the turbulent or wave motions at the surface and is probably governed by the wave-induced cyclic compression of the suspension [*Wadhams*, 2000]. The cakes will continually crash into neighboring cakes, and the cakes will become round. The freezing continues along the cake edges and eventually the pancakes will reach a diameter of 3-5 m and 50-70 cm thickness, see Fig. (2.1) and (2.2) [*Doble, et al.*, 2003; *Doble and Wadhams*, 2006; *Wilkinson*, 2006]. Furthermore, strong wave action may force the pancakes to a thick layer where the pancakes float on top of each other creating a thick layer of pancake slush [*Dai, et al.*, 2004]. It should be noted that the presence of pancake ice often implies that sea water is abundant all the way to the surface, with high temperatures at the surface, and subsequent very high heat losses from the surface. Once more solid ice forms the rate of ice freezing becomes much lower [*Wadhams, et al.*, 1987].

There are some evidences that it is mainly the wave action that prevents the pancakes to become attached to each other. In areas with large wave amplitudes such as the outer rim of the Antarctic sea ice and in the Greenland Sea there are large areas of pancake ice [*Wadhams*, 2000; *Wilkinson*, 2006]. However, it is also notable that pancake ice damp waves such that if the area with pancake ice is large enough solid ice will start to form in the calmest part of the pancake ice area: This is clearly seen around the Antarctic sea ice where the pancake zone can be up to 270 km wide [*Doble, et al.*, 2003; *Doble and Wadhams*, 2006; *Wadhams*, 2000; *Wadhams, et al.*, 1987].

Figure 2.1 Typical pancake field in the outer ice region: Left panel is from Antarctica [*Wadhams, et al.*, 1987] and right panel is from the Odden ice tongue in the Greenland Sea, here the pancakes are about 1-2 m (the stick on lower left is 1.5 m) [*Wadhams, et al.*, 2002]

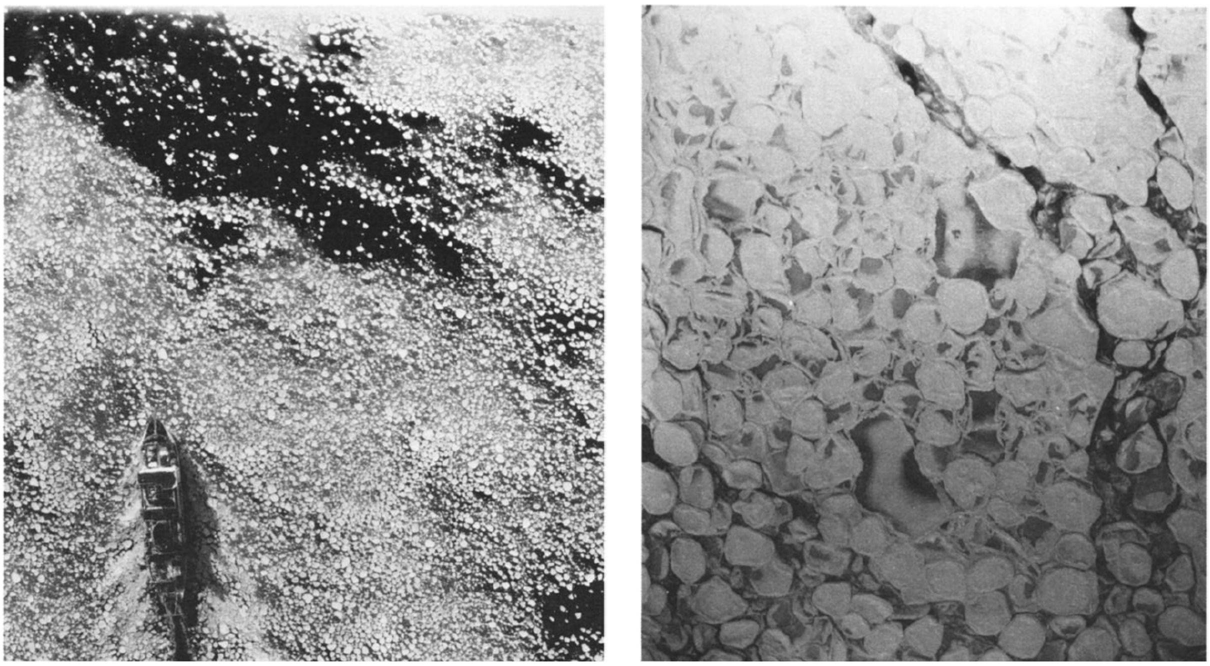

Figure 2.2 Typical pancake field in the outer ice region [*Wadhams, et al.*, 1987]. The picture on the right panel has a side of 35 m and the pancakes are about 2-4 m wide.

2.1.1.3 Solid ice

In cold calm conditions the pancake ice will start to form a homogenous solid pack ice: Of some interest for wave modeling purposes is that the solid ice that forms from pancake ice can be very rough with a roughness of say, 3 times the mean ice thickness, under intense wave conditions [*Wadhams, et al.*, 1987]. However, in areas with a less intensive wave climate such as in the Arctic Mediterranean the ice will be smoother. Furthermore, the thermodynamic growth and melting process tends to even out in-homogeneities such that ice that spent time in calm areas tend to be relatively homogenous [*Wadhams*, 2000].

For solid ice there is usually a distinction between first year ice (created this season) and multi-year ice (created at least the last season). As the freezing process is slow, solid ice with thickness over one meter tends to be multiyear ice. First year ice also tends to be saltier than the multiyear ice (the ice is slowly drained of salt), which implies that first year ice may have different mechanical properties than multi-year ice.

The solid ice is not homogeneous in any way, and there are a large number of discontinuities in the pack ice. In situations with divergent wind stress the ice will split up displaying open water areas (or leads) within the pack ice. These areas will start to freeze and we accordingly find areas with thin ice cover. There are also small areas with thick ice (peculiar enough, thick ice stems from areas with thin ice cover that has been crashed by ice movements); these are called pressure ridges and they have a keel that is about 4 times as deep as the sail (they are wider below the water interface than above). The pressure ridges can be over 50 m thick although most ridges are about 10-30 m thick [*Wadhams*, 1981; 2000; *Wadhams, et al.*, 1987; *Wadhams, et al.*, 2006]. However, again it should be noted that the freezing/thawing process in itself tends to make the ice cover more homogenous; it is some kind of mechanical actions that makes the ice cover inhomogeneous. Near the shore the ice coverage may extend all the way onto the land; and this type of ice is called land-fast ice [*Divine, et al.*, 2003]. It should be noted that changes in ice thickness are associated with changes in waves speed, and leads and pressure ridges will thus affect the wave propagation in solid ice.

2.1.1.4 Scattered floes and the Marginal Ice Zone (MIZ)

At some circumstances, for instance during intensive wave action or during divergent wind situations, the solid ice will breaks up into small pieces with sizes ranging from a few meters to several hundred meters. In most cases the ice floes will appear in the vicinity of the pack ice and the area with the ice floes are named the Marginal Ice Zone (MIZ). The MIZ is most characteristic in areas with strong wave action such as the Antarctica, Bering Sea, Greenland Sea, and to some extent the Barents Sea [*Wadhams*, 2000]. Some of the characteristic of the MIZ is displayed in Figures (2.3 and 2.4).

The MIZ is highly variable and represent a zone where waves, ice and atmospheric forcing interacts in a complicated way. Under strong wave conditions the wave can break up ice such that the MIZ becomes 70 km thick whereas it may be essentially vanishing under calm conditions [*Wadhams*, 2000]. Furthermore, the MIZ can move more that 50 km during a single day [*Perrie and Hu*, 1996], mainly as a response to wind forcing but wave forcing and ocean currents can also play important roles. Here it should be noted that waves impinging on a MIZ will force the ice floes towards the ice margin by the waves radiation stress [*Longuet-Higgins*, 1977; *Perrie and Hu*, 1997; *Wadhams*, 2000]. In situations where wind is blowing over the MIZ towards the open ocean ice will drift towards the sea in well-defined bands parallel to the ice edge, the waves generated by wind between the bands will put a pressure on the back side of the ice band (the side towards the MIZ and the wind generated waves) thus keeping the band together [*Wadhams*, 1983; 2000]. It should be noted that the MIZ often respond very differently to on-ice wind as compared to off-ice wind. As a final notice on the importance of ice for the MIZ we notice that severe wave forcing may in principle pulverize the ice creating a slush ice layer [*Frankenstein, et al.*, 2001]. In addition to the large scale wind pattern there may also be local wind cells driven by the contrast in air temperature over ice and over water [*Chu*, 1987].

We may conclude that the dynamics of the MIZ and the wave characteristics in the MIZ is very difficult to model. This is unfortunate as this zone that is probably the most interesting zone for an operational wave-ice model.

There are some valuable observations on the size distribution and geometry of the ice floes in the MIZ. Some of these observations suggest that there is a zonal distribution of ice floes, with small floes in the outer MIZ, a flat medium size distribution in the interior zone and large floes near the solid ice edge [*Lu, et al.*, 2008; *Squire and Moore*, 1980]. It has been suggests that the distribution of ice floe size and geometry follow some type of distribution law [*Lu, et al.*, 2008], i.e.,

$$\frac{N(<L)}{N_0} = 1 - \exp\left[-\left(\frac{L}{L_0}\right)^{\gamma}\right], \tag{2.1}$$

where N is the cumulative number of floes with size smaller than L; N_0 is the total number of floes and L_0 (scale coefficient) and γ (shape coefficient) are two parameters that can be fitted to data. The value of these observations for wave-in-ice studies has not been utilised to any major extent.

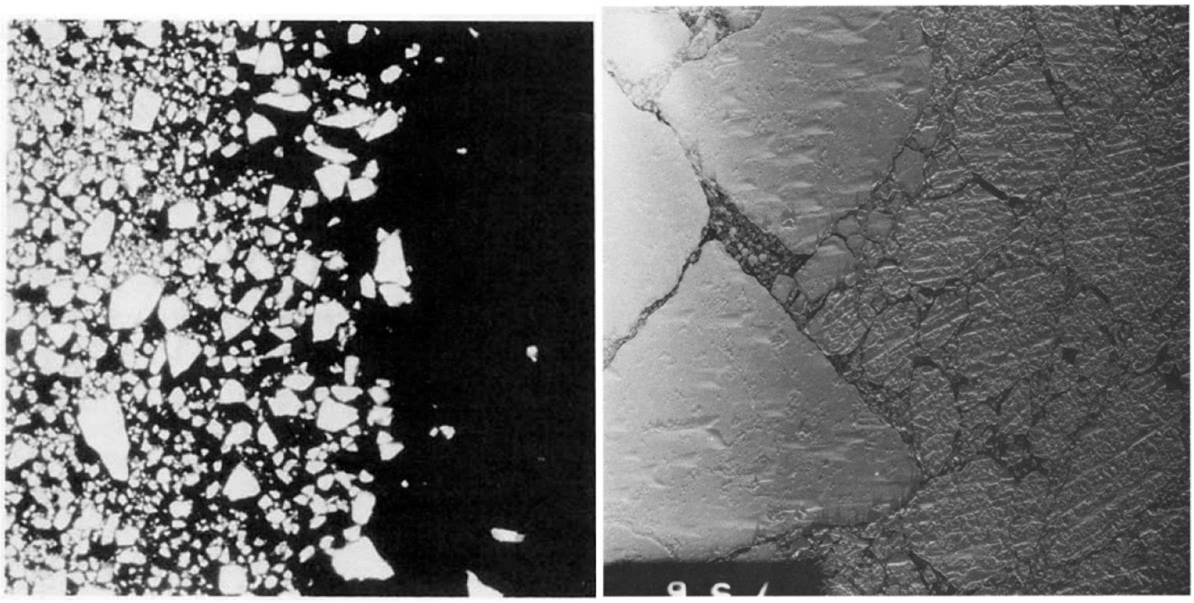

Figure 2.3 Solid ice field that have been broken up by swell from a distance storm, Left panel is from the ice edge during the MIZEX experiment in the Greenland Sea 1984 [*Wadhams, et al.*, 1986]. The right panel is from Antarctica, the size of the picture is 300 m [*Wadhams, et al.*, 1987]

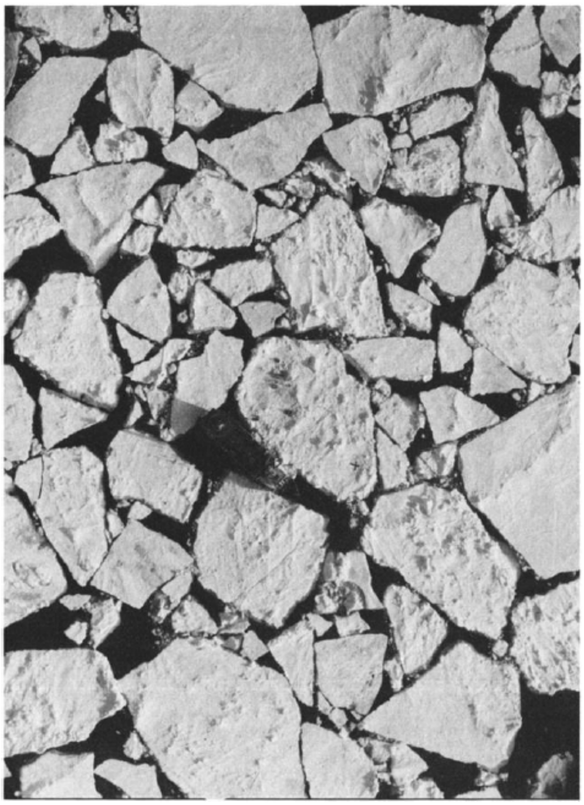

Figure 2.4. Aerial photograph of Greenland Sea MIZ sea ice, with the 50-m-long ship Polarbjørn near the center. The photograph was taken during the 1983 Marginal Ice Zone Experiment [*Meylan and Squire*, 1994].

2.1.2 Observations on wave characteristics in ice

Let us consider a typical situation that may appear in a real case scenario. Wave and ice conditions are very variable but some characterization is possible; however, it should be remembered that there are many exceptions. Starting from the open ocean and moving into the ice we have the following areas where each area have different physical characteristics with respect to wave dynamics [*Lu, et al.*, 2008; *Squire and Moore*, 1980; *Wadhams*, 2000]:

- **Open ocean:** At some distance from the ice edge we will have open ocean water, and waves will have typical open water characteristics.
- **Edge zone** with scattered ice floes (i.e., the outer zone less than 5 km from open ocean): In the outer MIZ region there are relatively few floes such that the floes do not interact with each other. When wave reaches an ice floe the floe movement will become a part of the wave motion. If the floe is large enough, i.e., it is of same size, or larger, than the wave length, the wave propagation will be altered due to the flexural-gravity forces of the ice floe (here both the stiffness and the added mass of the ice are considered, see Appendix B). The wave velocity accordingly changes and these changes are associated with a certain scatter of the wave energy according to Snell's law. Waves tend to break large ice floes into smaller pieces and therefore only small floes are found in the edge zone in cases with energetic waves.
- **Transition zone** has frequent ice floes of intermediate size. The transition zone is characterized by being rather homogenous with respect to ice conditions. The breaking of ice floes into smaller floes exist but is it does not have an overall importance in the transition zone. The scattering of waves due to the wave ice-floe interaction are important; however, the high frequency of ice floes implies that the floe-floe interaction, or collisions, also becomes very important, or even dominant [*Squire, et al.*, 1995]. It has been shown that the rate of the collision depends on the floe concentration and to some extent on the wave amplitude [*Squire, et al.*, 1995]. The interface between the transition zone and the outer edge is often characterized by the breaking of ice floes into smaller pieces due to the action of waves. As of today there is no clear and homogenous picture of the role of floe floe collisions and how the rate of the collisions depends on external forcing. It is still an open question how well the "standard" wave scattering model (Appendix B, Sec. 2.3.1) will describe the dynamics of this region (one fundamental assumption in the scattering model is the floe-floe interaction is negligible).
- **Interior MIZ**. The interior zone is closest to the solid pack ice and the largest ice floes are found here. The waves that appears here has very weak amplitude (as the wave energy has been scattered as the waves travel towards the inner zone) and have long wavelength (shorter waves are scattered more efficient implying that only the long waves can penetrate deep into the MIZ). If more energetic waves enter the interior zone, the ice floes start to break up. However, the waves will also create new ice floes by breaking up the solid pack ice upholding the interior MIZ zone.
- **Solid ice.** In solid ice waves tend to propagate almost unperturbed and can travel long distances. The major dissipation of wave energy comes from scattering due to leads and pressure ridges (which alters the wave speed through the flexural-gravity forces), viscous dissipation at the wave-water interface, and creep within the ice [*Liu and Mollo-Christensen*, 1988; *Wadhams*, 1973]. The attenuation of waves under solid ice has not received much attention as compared to the wave ice-floe scattering process.

In addition to these categories we also have wave propagation in the following "media"

- **Grease ice:** Grease ice is very viscous and frictional affects become important for describing the wave propagation. As the grease ice float on the top of the ocean this system can be viewed as a two-layer system with a high-viscosity grease-ice slush on top of a low viscosity water layer [*Weber*, 1987].
- **Slush ice:** During strong wave conditions the ice floes are rapidly broken into small pieces, and these pieces constantly crush into each other. The result is a mixture of small ice floes and small ice particles dissolved in the water, and can thus be considered as slush ice [*Frankenstein, et al.*, 2001]. Thus, small waves (with size corresponding to the size of the pancakes) with be affected by scattering processes: Large waves will not be affected by scattering and it seems reasonable that there are some strong similarities with wave propagation in grease ice and in dense pancake ice (i.e., wave propagation in very viscous two-layer fluid).
- **Pancake ice:** Pancake ice fall in-between grease ice and ice with dense floe concentration. To some extent it is also close to slush ice as described above. Observations in pancake ice indicate the waves passing through dense fields of pancake ice has certain similarities with waves passing through grease ice [*De Carolis and Desiderio*, 2002; *Fox and Haskell*, 2001; *Wadhams, et al.*, 2002; *Weber*, 1987].

2.1.3 Attenuation of waves in ice

The energy of the waves will in general decay exponentially from the edge of the MIZ and inwards to the solid ice. The main conclusions are [*Dixon and Squire*, 2001; *Fox and Haskell*, 2001; *Fox and Squire*, 1990; *Frankenstein, et al.*, 2001; *LaRouche and Cariou*, 1992; *Meylan and Squire*, 1994; *Perrie and Hu*, 1996; *Schulz-Stellenfleth and Lehner*, 2002; *Squire*, 1984; 1995; 2007; *Squire, et al.*, 1995; *Squire and Moore*, 1980; *Wadhams*, 1973; 2000; *Wadhams, et al.*, 2002; *Wadhams, et al.*, 2004; *Wadhams, et al.*, 1986; *Wadhams, et al.*, 1988]

- The scattering and the dissipation of wave energy depends on the energy of the wave.
- The scattering at the outer ice edge is normally a few percent, even in cases with a compact ice edge.
- The wave energy decays exponentially with an attenuation coefficient that varies between 2×10^{-4} m^{-1} for long waves to 8×10^{-4} m^{-1} for 8-9 s waves, correspond to e-folding distances of about 5-1.2 km [*Wadhams*, 2000; *Wadhams, et al.*, 1988]. These value values are based on "mean" values and it should be noted that there is great uncertainties about the exact values of the attenuation coefficient and how it depend on the ice state [*Frankenstein, et al.*, 2001; *Squire and Moore*, 1980].
- The attenuation rate may depend on the temperature (due to the mechanical properties of ice near the melting point) and very high attenuation rates (4-1×10^{-3} m^{-1} for wave periods of 6 to 12 sec) was obtained for near melting temperatures [*Squire*, 1984].
- There is a "roll-over" effect for the shortest wave periods (say 6-8 s), where wave damping is weak. The exact reason for the roll-over effect remains unclear (it has been suggested that these short waves are created by the ice-floe movements, by local wind generation, etc.).
- The scattering process reflects waves in many directions due to the irregularity of the floe shapes. The result is that the original uni-directional waves are scattered such the wave field becomes omi-directional at some distance into the ice field [*Schulz-*

Stellenfleth and Lehner, 2002; *Wadhams, et al.*, 1986]. Short wave are quickly scattered and the spectra initially tends towards the longer uni-directional swell; however, deeper into the MIZ the scattering process takes over and a more homogenous wave spectra is observed.

- For the simple case when the floe is much larger than the wave length and assuming that there are no floe-floe interactions and that the floe edge is perpendicular to the incoming wave direction it is reasonable to assume that each floe scatters a certain fraction of the incoming wave energy [*Kohout and Meylan*, 2008a].
- Frictional affect are important. Wave energy is continuously pumped into the MIZ; albeit scattering process will change the direction of propagation, the total energy is conserved. By observation we know that the wave energy diminishes as we travel into the MIZ implying that there must be a significant damping of the wave field by viscous processes [*Wadhams, et al.*, 1988].
- There may be a very strong attenuation of waves in slush ice [*Frankenstein, et al.*, 2001], while the attenuation is relatively weak for swell in pancake ice [*Wadhams, et al.*, 2002; *Wadhams, et al.*, 2004].

The attenuation of wave energy depend on the energy itself such that we can write [*Wadhams*, 2000; *Wadhams, et al.*, 1988]

$$\frac{\partial E}{\partial x} = -\alpha E \tag{2.2}$$

where α is the attenuation coefficient. This equation has the solution

$$E = E_0 e^{-\alpha x} \tag{2.3}$$

where E_0 is the energy of the waves at x=0 (i.e., the ice edge). The attenuation coefficient depends on the scattering process of each ice floe that is encountered; for a specific ice-floe class it can be written

$$\alpha_i = \frac{p_i r_i}{d_i} \tag{2.4}$$

where α_i, p_i, r_i, d_i, are the scattering coefficient, fractional coverage, energy reflectivity, and diameter of the ice class i respectively. Taking the sum of all ice floes classes and their scattering efficiency we may write [*Wadhams*, 2000; *Wadhams, et al.*, 1988]

$$\alpha = \sum \alpha_i \tag{2.5}$$

Another formulation is that the scattering depends on the number of ice floes that are encountered such that

$$E = E_0 e^{-na} \tag{2.6}$$

where n is the number of floes and a is the scatter coefficient for each floe [*Kohout and Meylan*, 2008a]. The basis of this model is the theoretical predictions that the scattering depends linearly on the number of ice floes that are encountered. This confirms the hypothesis above that the scattering from each ice class can be summarized to obtain an overall scattering coefficient (i.e., Eq. 2.5). Let us assume that a class of ice floes will have a scattering of 4 % (see Fig. 2.9 where it should be remembered that energy scattering is the square of R shown in the figure) after encountering 25 ice floes the energy have decreased by e^{-1}, or about 80%. If we assume that the floes are 100 m wide and the concentration is 50% we find an exponential length scale for wave energy decay of 5 km, in fair agreement with observations.

2.1.4 Ice in wave fields

For many of the theories describing the wave dynamics under ice condition the ice is either considered as solid or as slush, which both are considered to be horizontal homogenous. This is of course an oversimplification although it is useful for mathematical purposes. There have not been many studies outlining the importance of floe floe interaction for its impact on the wave scattering and energy dissipation. However, it is clear the floe floe interactions are important and is easily observed in field experiments [*Fox and Haskell*, 2001; *Martin and Becker*, 1987; 1988; *Martin and Drueker*, 1991; *McKenna and Crocker*, 1992; *Rottier*, 1992; *Squire*, 2007; *Wadhams*, 2000].

Moving ice floes have six degrees of freedom for rigid-body movements: three types of movement, surge, heave, and sway; and three types of rotations, pitch, roll and yaw. All these movements can be studied using accelerometers. Observations using accelerometers show that vertical movements are much stronger than the horizontal accelerations. However, there are certain strong spikes in the horizontal acceleration that can be interpreted as collisions of the ice floes with surrounding ice floes. On average, each collision event appears to be associated with about 0.01 m displacement over a 10 s period, giving a characteristic velocity of about 0.001 m/s. Each of these collisions will create some ice crushing (although the collisions may be damped when the concentration of small ice particles become high). The collision rate is about a few per minutes [*McKenna and Crocker*, 1992; *Rottier*, 1992] but may vary greatly with wave and ice conditions; the collision rate appears to be difficult to relate to observed parameters making modeling of the occurrence troublesome. However, using basic physical arguments it is reasonable to assume that strong wave fields will create stronger impacts, and probably also more frequent collisions. The concentration of ice floes is of course also an important parameter. It is likely that a strong wave field will create large concentrations of very small ice particles such that the water will take some resemblance on slush ice [*Frankenstein, et al.*, 2001].

2.1.4 Observations on dispersion/wave length

The dispersion relation describes the relation between the wave period and the wavelength, if these quantities are known the phase speed and the group velocities can be calculated. Furthermore, the dispersion relation depends on the type of ice that is present and the ice thickness. It is not simple to make observations on the dispersion relation under sea ice; nevertheless, there exist certain observations [*Fox and Haskell*, 2001; *Liu and Mollo-Christensen*, 1988; *Schulz-Stellenfleth and Lehner*, 2002; *Wadhams, et al.*, 2002; *Wadhams, et al.*, 2004].

Fox and Haskell made observations using accelerometers placed on small, say 5 m diameter, ice floes [*Fox and Haskell*, 2001]. One of their main results were that that the measured wave-number was consistently larger (i.e., shorter wave-lengths) than those estimated for open ocean conditions: The best fit was given by the function $k \propto \omega^{2.41}$. These observations are not consistent with the added mass theory (see Appendix B1.2) or the flexural gravity wave finding, which predicts that the wave number should be lower in ice covered areas as compared to open water conditions.

Wadhams et al. [2002] made some estimates on the wave dispersion in frazil-pancake ice-fields using SAR images. One of the results from this study is that the estimated wave-number again were consistently larger (i.e., shorter wave-lengths) under ice conditions than estimates for open ocean conditions. This is not consistent with the frequently used mass-load model to describe waves in pancake ice (see Appendix B and Eq. B 17 b). Wadhams et al [2002] also conclude that the amplitude is smaller under the sea ice than for the open ocean conditions. When attempting to invoke the ice thickness from observed changes in wave properties, too high ice thicknesses were obtain using the mass loading theory (i.e., estimated thickness of 2.1 m but observed thickness was 50 cm). A comparison with a viscous slush ice model [*Weber*, 1987] gave somewhat more consistent results albeit a very high viscosity had to be used to fit model toward data. It was noted that the unknown nature of the viscosity that has to be used within this model implies that it is probably difficult to invoke ice thickness from observations on wave variables using this model. One note of interest is that for the one case where the waves passed through the ice field into open water, the wave regained its original wave amplitude. Thus, the reduction in wave height seems to be coupled to the presence of frazil-pancake ice.

Liu and Mollo-Christensen [1988] did not make direct measurement on the dispersion relation, but they did estimate the relation between wave frequency and wave length for a case with intensive wave field deep inside the Antarctic pack ice. The conclusion was that the observed dispersion follow predictions from a flexural-gravity model based on Bernoulli-Euler thin plate theory (see Appendix B). Furthermore, the authors presented convincing evidence that the group velocity became very slow due to the flexural ice cover (which implies that the wave amplitude increases, Appendix A), and this could explain the sudden appearance of energetic waves deep inside the pack ice. After the solid ice cover was ruptured by the waves, the dispersion relation could be explained by the dispersion relation typical for open water theory.

2.2 Laboratory work

It is often difficult to make detailed observations during field studies, and it also is difficult to control the conditions. Laboratory experiment thus provides a fruitful way to study the system under controlled conditions and using dedicated probes that cannot be used in the field. A caveat of laboratory work is that many of the natural conditions cannot be simulated; the problem with replicating the size of the natural problem is the most severe shortcoming of the laboratory experiments. Nevertheless, laboratory studies of mainly slush, or grease, ice has been important to advance our understanding of wave and ice interaction.

2.2.1 Slush ice experiments

Slush ice can be created by using a wave tank in a cold-room. By continuously creating waves and keeping the room temperature at, say, -10^{o}C, a homogenous slush ice layer with small ice disc (order mm large) are formed. A typical experimental setup is shown in Fig. (2.5). The most obvious observations in these types of experiments are [*Martin and Kauffman*, 1981; *Newyear and Martin*, 1997]:

- It is clear that the wave action suppresses the sintering of ice crystals into larges ice objects confirming the hypothesis that waves are important for maintaining the slush ice.
- The slush layer is homogenous and has a clear vertical extent; furthermore, the wave forces the slush ice towards the opposite end of the wave tank as the result of the induced wave drift and the radiation stress exerted by the wave field [*Martin and Kauffman*, 1981]. Here it may be noted that waves carries a certain momentum and

when the wave amplitude decreases the wave momentum will also decrease: However, momentum is a conserved quantity and the access momentum will affect the surroundings as a certain radiation stress [*Longuet-Higgins and Stewart*, 1960; 1964].

- The waves are quickly damped by the slush ice, and the wave amplitude declines exponentially with distance from the wave generator. The observed wave amplitude and the thickness of the slush layer can be modeled using a two-layer model where the upper layer represents the buoyant and viscous slush ice layer whereas the bottom layer is ordinary sea water [*De Carolis and Desiderio*, 2002; *Newyear and Martin*, 1997; *Weber*, 1987] (See also Sec. 2.3 and Appendix C). All studies clearly show that frictional affects must be included to explain laboratory experiments using slush ice.

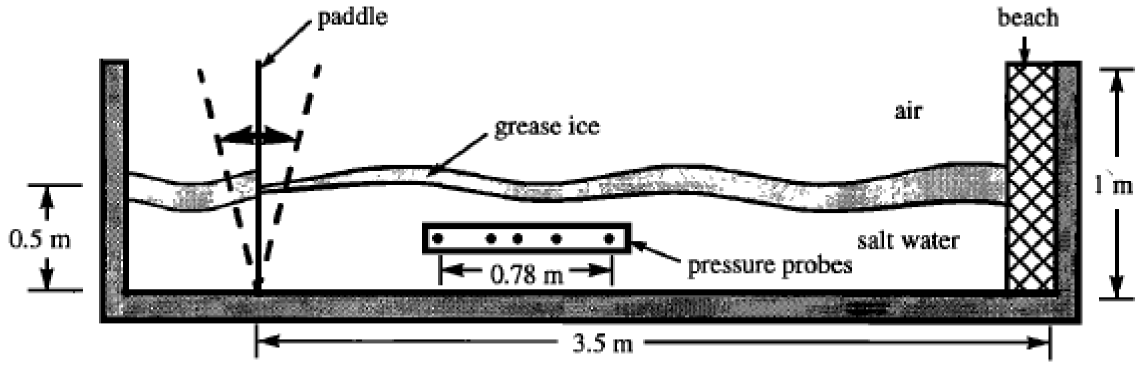

Fig 2.5 A typical laboratory experiment for studying the wave-slush ice interaction [*Newyear and Martin*, 1997]. Note that the wave forces the slush ice away from the wave generator such that a thicker slush ice layer is found at the other end of the tank.

2.2.2 Experiment with solid obstacles

It is expected that small floating objects to some extent will behave as buoyant water-object mixture: the described scenario accordingly show some resemblance to the situation with small pancake ice in the sea. Observations show that ice pancakes may float on top of each other in wave conditions [*Dai, et al.*, 2004]. Laboratory experiment with small and thin plastic plates confirms the hypothesis that the waves may force the pancakes on top of each other [*Dai, et al.*, 2004]. Visual observations of the thickness of ice-object mixture and the rate of wave damping shows that this type of experiments have similarities with the experiments using slush ice described above.

2.2.3 Experiment with solid ice

When waves encounter solid ice there will be a certain reflection of the wave energy. Furthermore, the wave will propagate as flexural gravity waves under the ice cover. The strain induced in the ice will break the ice if the strain is large enough. Some experiments to verify the theoretical predictions have been considered, qualitative agreements were found but due to some experimental problems a thorough comparison between experiments and theory could not be done [*Squire*, 1984].

2.3 Theoretical studies on wave-in-ice

During this study it has become clear that there exist a very large literature on the theoretical aspects of waves in ice conditions. The main focus has been on deriving mathematical methods for describing the wave scattering from ice floes. There are also some works on describing the situation with high-viscosity slush-ice on top of ordinary sea water. Here we will only describe the models briefly but a more thorough derivation of the equations can be found in Appendix B (for the scattering model) and Appendix C (for the two layer slush-ice sea-water system. Here a very brief description of the scattering models are give and more compete pictures can be found in review articles [*Squire*, 2007; *Squire, et al.*, 1995].

2.3.1 Studies of scattering processes from sea ice

Here we give a brief description of the commonly used wave ice-floe scattering model. A more detailed derivation can be found in Appendix B. Let us consider a situation where open water waves enter from left (i.e., from x=-∞) and that they encounter an ice floe extending from x=0 to x=L having thickness h. The wave will propagate under the ice cover: However, as the wave speed is different under the ice cover there will be a certain reflection of the wave energy due to the ice cover, and only a certain part of the wave energy will propagate through the ice cover over to the open water at the other side of the ice floe. An important assumption in this theory is that there are no floe floe interactions. A schematic picture of the situation is outlined in Fig. (2.6).

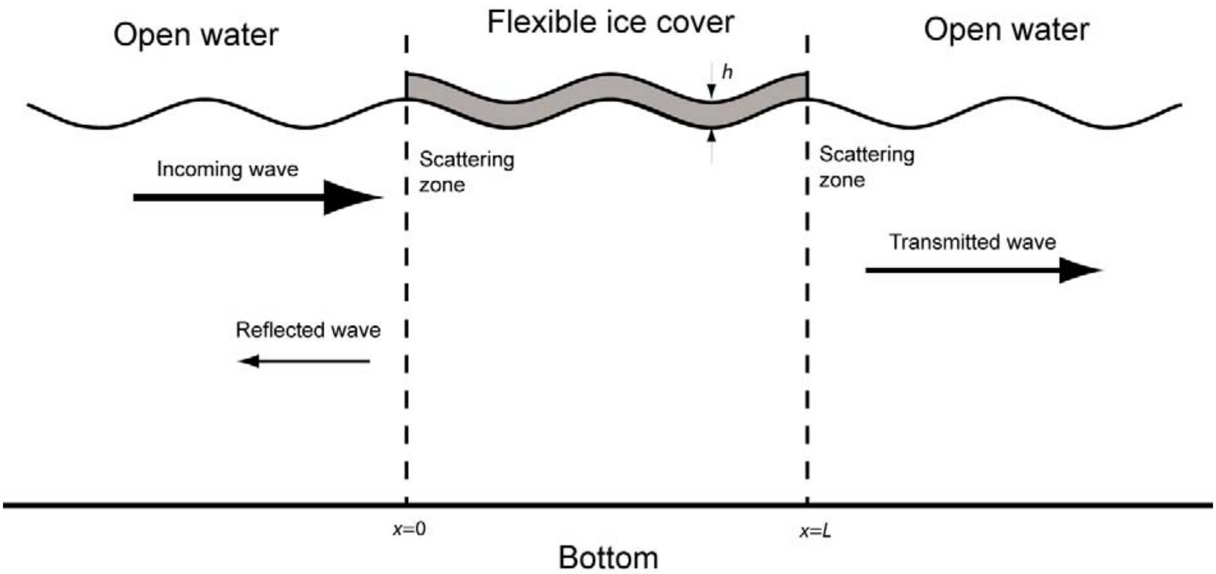

Figure 2.6 Schematic picture of the physical situation when a wave encounters an ice floe.

2.3.1.1 Outline of the most frequently used mathematical model

Here we will assume that the wave field can be described through a velocity potential $\phi(x,z,t)$ such that $u = \partial\phi/\partial x$, $w = \partial\phi/\partial y$: The equation for the interior of the fluid is given by

$$\nabla^2\phi = 0\,. \qquad (2.7)$$

Surface boundary conditions are (assuming p=0 at the surface)

$$\rho_w g \frac{\partial\phi}{\partial z} + \rho_w \omega^2 \phi = 0 \quad \text{open water,} \qquad (2.8a)$$

$$\left(L\frac{\partial^4}{\partial x^4} + \rho_i h \omega^2 + \rho_w g \right)\frac{\partial\phi}{\partial z} + \rho_w \omega^2 \phi = 0 \quad \text{ice conditions.} \qquad (2.8b)$$

Here the first term in (2.8b) reflects the flexural strength of ice, while the second term describes the added mass due to the ice cover. Constants are

$L = \dfrac{Eh^3}{12(1-s^2)}$, which is the flexural rigidity of ice, where E=6·10^9 N m^{-2} is the Young's modulus of elasticity for ice, and s=0.3 is the Poisson's ratio for ice.

ρ_i is the density of the ice, and the second term represents the added mass due to ice.

Matching conditions at the edge of the ice floe are (the bending moment and the shear must vanish and there must be a continuity in the velocity potential)

$$\left.\frac{\partial^2\phi}{\partial x^2\partial z}\right|_{\substack{x=x\\ z=0_i}} = 0,\quad \left.\frac{\partial^3\phi}{\partial x^3\partial z}\right|_{\substack{x=x_i\\ z=0}} = 0\,, \qquad \text{(bending moment)}, \qquad (2.9a, b)$$

$$\phi^+\big|_{x=x_i} = \phi^-\big|_{x=x_i}\,,\quad \left.\frac{\partial\phi^+}{\partial x}\right|_{x=x_i} = \left.\frac{\partial\phi^-}{\partial x}\right|_{x=x_i}. \qquad \text{(continuity)}, \qquad (2.10a, b)$$

Bottom boundary condition reads

$$\left.\frac{\partial\phi}{\partial z}\right|_{z=-H} = 0\,. \qquad (2.11)$$

Equations (2.8)-(2.1) represent the "standard" set of equations used to describe the scattering of waves that encounter an ice floe. A schematic view of the equations and the boundary and matching conditions are outlined in Fig. (2.7).

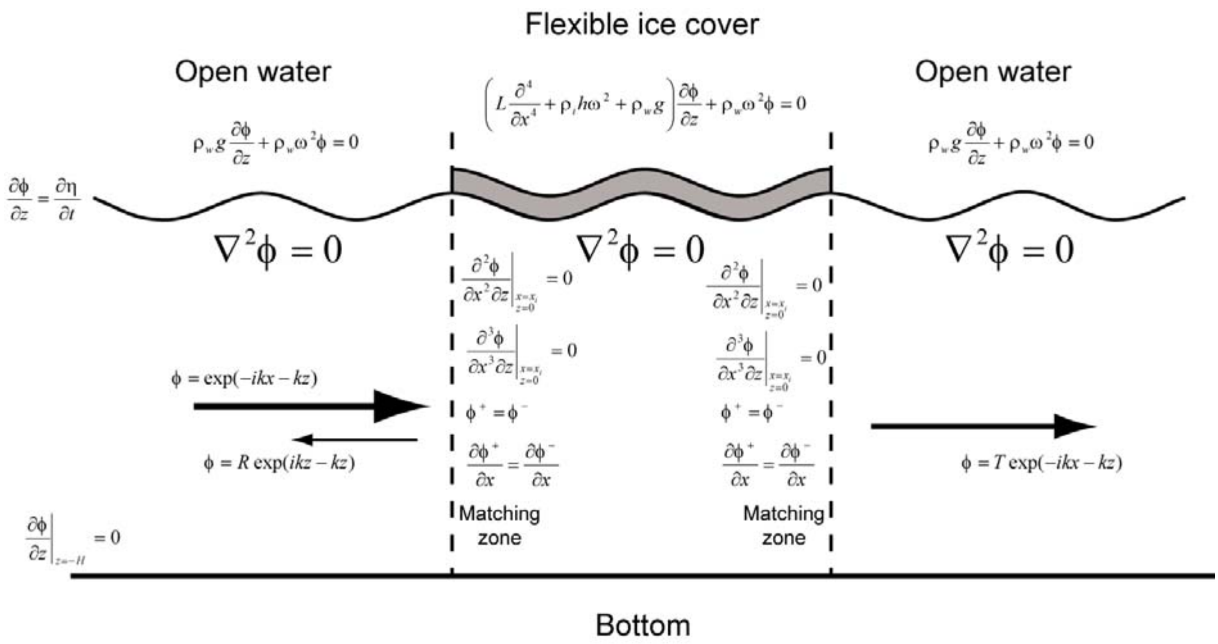

Figure 2.7 A schematic view of the equation and boundary and matching conditions that has to be applied for deriving the wave field when waves encounter an ice floe. The figure also shows the schematic solution where an incoming wave is scattered by the ice floe.

The methods used to solve the above set of equations are rather complex. The first solutions were sketched in 1968 using the Weiner-Hopf method [*Evans and Davies*, 1968] but this solution could not be used in a practical way; the first applicable approximate solutions were derived in early seventies [*Wadhams*, 1973], although this solution have been criticized for being too simple [*Meylan and Squire*, 1994; *Squire, et al.*, 1995]. Exact solutions (albeit numerical to some degree) were derived in the early 1990'ties [*Fox and Squire*, 1990; 1991b; 1994; *Meylan and Squire*, 1993; *Meylan and Squire*, 1994]. A wide range of mathematical methods have been deployed involving methods from complex variable theory such as the Wiener–Hopf method, Fourier analysis using variational (numerical) methods of estimating matching constants [*Fox and Squire*, 1990; 1991a], residue calculus, Green's functions and integral equations [*Meylan and Squire*, 1993; *Meylan*, 2002; *Meylan and Squire*, 1994], Rayligth-Ritz method in combination with variational calculus [*Bennetts, et al.*, 2007], eigenfunction expansions [*Kohout and Meylan*, 2008a; *Kohout, et al.*, 2007], and by numerical methods [*Lavrenov and Novakov*, 2000]. None of these models are, however, easily accessible and have not been used in combination with numerical wave modeling.

During the last few decades here has been a very intensive development on the mathematical treatment of the problem outlined above [*Squire*, 2007]. The latest developments are mainly focusing on

- Mathematical models of multiple ice floes [*Kohout and Meylan*, 2008a; *Kohout and Meylan*, 2008b; *Kohout, et al.*, 2007].
- Description of in-homogeneities in the ice floe thickness [*Bennetts, et al.*, 2007; *Squire*, 2007; *Squire and Dixon*, 2001; *Squire and Williams*, 2008; *Williams and Squire*, 2006; 2008].

Again it should be emphasized that many of these studies use advance mathematical methods that are not easy to access for non-mathematicians.

Albeit there have been an intense research on the mathematical modeling of wave ice-floe interaction, there have not been the same intensity in experimental and observational research. Thus, today it is stated that the absence of observational and experimental data is the main obstacle for advancing the research on wave ice-floe interaction [*Kohout and Meylan*, 2008a; *Squire*, 2007].

2.3.1.2 Sketching one possible solution

The solution to the mathematical problem outlined (2.7-2.11) is not straightforward: However using Green functions the solution may be written as [*Meylan and Squire*, 1994]

$$\begin{aligned} \phi &= \exp(-ikx - kz) + R\exp(ikz - kz), x \to -\infty, \\ \phi &= T\exp(-ikx - kz) \qquad , x \to \infty, \end{aligned} \tag{2.12}$$

The transmission and reflection coefficients are complex integral functions that can be written

$$R = ik\int_0^L e^{-ik\xi}\left[\phi(\xi,0)+\beta\int_0^L g(\xi,\varsigma)\phi(\varsigma,0)d\varsigma\right]d\xi,$$

$$T = 1+ik\int_0^L e^{ik\xi}\left[\phi(\xi,0)+\beta\int_0^L g(\xi,\varsigma)\phi(\varsigma,0)d\varsigma\right]d\xi, \qquad (2.13a, b)$$

where

$$\phi(x,0) = e^{ikx} + k\int_0^L G(\xi,0;x,0)+i\cos[k(\xi-x)]\left[\phi(\xi,0)+\beta\int_0^L g(\xi,\varsigma)\phi(\varsigma,0)d\varsigma\right]d\xi\,. \qquad (2.14)$$

Here, g is the Green function representative of the boundary condition (2.9 b)

$$\frac{d^4 g(\xi,x)}{d\xi^4} - \delta^4 g(\xi,x) = \delta(\xi - x)\,, \qquad (2.15)$$

where δ is a non-dimensional parameter [*Meylan and Squire*, 1994]. The boundary conditions are

$$\frac{d^2 g(0,x)}{d\xi^2} = \frac{d^2 g(L,x)}{d\xi^2} = \frac{d^4 g(0,x)}{d\xi^4} = \frac{d^4 g(L,x)}{d\xi^4} = 0\,. \qquad (2.16)$$

This solution is non-trivial such that we have only sketched the solution in this study.

It may be noted that the solution outlined above is complex: furthermore, when considering multiple ice floes or in-homogeneities in the ice cover the complexity of the mathematics increases significantly [*Dixon and Squire*, 2000; 2001; *Kohout and Meylan*, 2008a; *Kohout and Meylan*, 2008b; *Kohout, et al.*, 2007; *Meylan and Masson*, 2006; *Squire*, 2007; *Squire and Williams*, 2008; *Williams and Squire*, 2006; 2008].

2.3.1.3 Energy transmission

Before taking a look on the solutions we recall that the main parameter we need to study is the transmission of energy, which is

$$t = |T|^2\,; \qquad (2.17)$$

furthermore we expect that (since energy is conserved in a scattering model)

$$|R|^2 + |T|^2 \equiv 1\,. \qquad (2.18)$$

As mention above, the solutions are not trivial and we have not derived any solutions our self in this study. Instead we have copied results from earlier studies to outline the structure of the transmission of energy. The transmission coefficient T as a function of the ice-floe length for different ice thicknesses is shown in Fig. (2.8), while the transmission is outlined as a function of wave frequency in Fig. (2.9). The figures shows some typical characteristics that is common among most theoretical models for scattering of waves by ice floes.

- There are a number of ice floes sizes that give zero reflection, or in other words a perfect transmission. This is probably a result of the simplified geometry of the outlined model [*Vaughan, et al.*, 2007] and in a more natural situation it is not likely that perfect transmission is important for the wave propagation in an ice field.
- The transmission increases as the ice gets thinner.

- Short waves (high frequency) tends the have higher reflectance than long waves (low frequency).

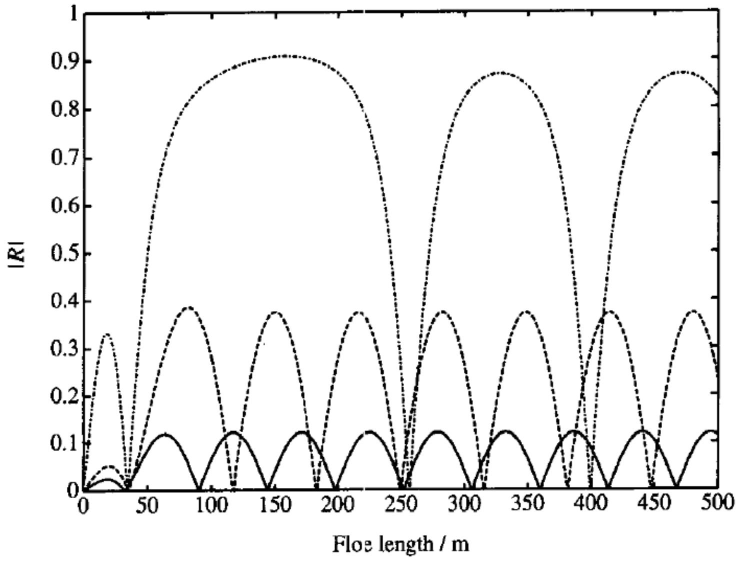

Figure 2.8 The reflection coefficient for different size of the ice floe [*Meylan and Squire*, 1994]. Physical parameters are h=0.5 m (solid curve), h=1 m (dashed curve), h=5 m (dash-dotted curve), and the wave length is 100m.

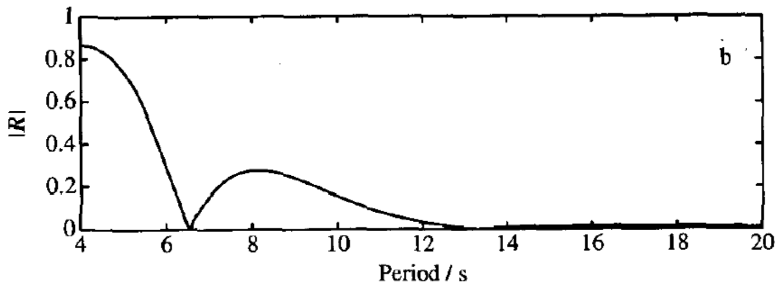

Figure 2.9 The reflection coefficient for different wave periods [*Meylan and Squire*, 1994]. Physical parameters are h=1 m and the size of the ice flow is 100m.

2.3.1.4 Ice strain

The strain that the waves impose on the ice is the main reason for the rapture of ice into smaller pieces. It is thus an important parameter for understanding how waves influence the ice [*Squire, et al.*, 1995]. The surface strain on the ice sheet, ε, can be estimated by the expression

$$\varepsilon(x, f) = \frac{h}{2}\frac{\partial^2 \eta}{\partial x^2} \tag{2.19}$$

The maximum strength as a function of the wave frequency is shown in Fig. (2.10-2.13). We see that short waves induces a strong strain in the ice, which is thus likely to break. We conclude that ice floes hit by short energetic waves will probably rapture very quickly; this explains the small ice floes observed in the edge zone of the MIZ. However, the short waves are attenuated quickly; since the longer remaining waves produces a significantly weaker strain the ice will not rapture after the shortest waves has dissipated. This explains the relatively constant conditions of the transition zone in the MIZ.

Wave period	Critical amplitude
5-10 s	0.09 m
15 s	0.28 m
>25 s	1 m

Table 1 The critical wave amplitude that will lead to a fracturing of the ice due to strain for an ice thickness of 1 m.

Some numbers on the critical amplitude that will cause a fracture in ice with a thickness of 1 m is provided in table 1. Wee see that the critical amplitude increases rapidly with wave period. The strain induced by the waves represents a complicated interaction between the incoming wave and evanescent scattering wave component. One result of this interaction among wave component is that the maximum strain rate lies some distance into the ice, Figure 2.10 [*Langhorne, et al.*, 1998; *Squire*, 1983; *Squire, et al.*, 1995]. The maximum strain as function of wave period is illustrated in Fig. (2.11), while the maximum strain and the position of the maximum strength is visualized in Fig. (2.12). The ice will probably break where the strain has the maximum and the ice floes that result from wave action is initially very homogenous (see Fig. 2.13). The process where wave break ice is very rapid as can be revealed from field campaigns for studying breakup of ice, where these campaign had to be abandoned under "controlled panic" before the measurements could be started [*Squire*, 1983; 1984].

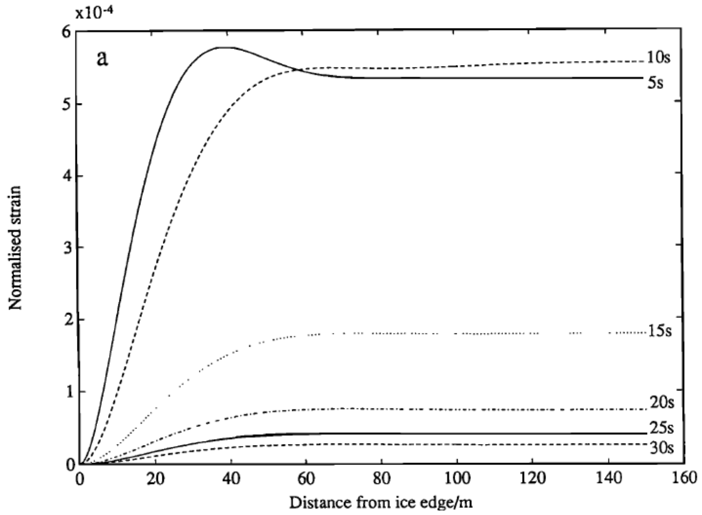

Figure 2.10 The maximal strain in the ice coefficient as a function of distance for the ice edge. The water depth is 100 m and the figure shows model prediction for a 1 m thick ice for different wave periods [*Fox and Squire*, 1991b]. Note that the maximum strain appears at some distance into the ice.

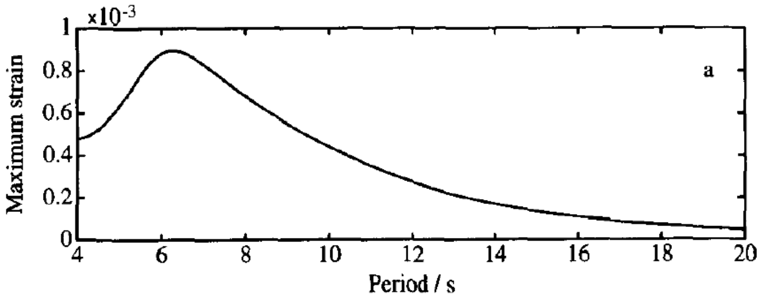

Figure 2.11 The maximal strain in the ice coefficient for different wave periods [*Meylan and Squire*, 1994]. Physical parameters are *h*=1 m and the size of the ice flow is 100m.

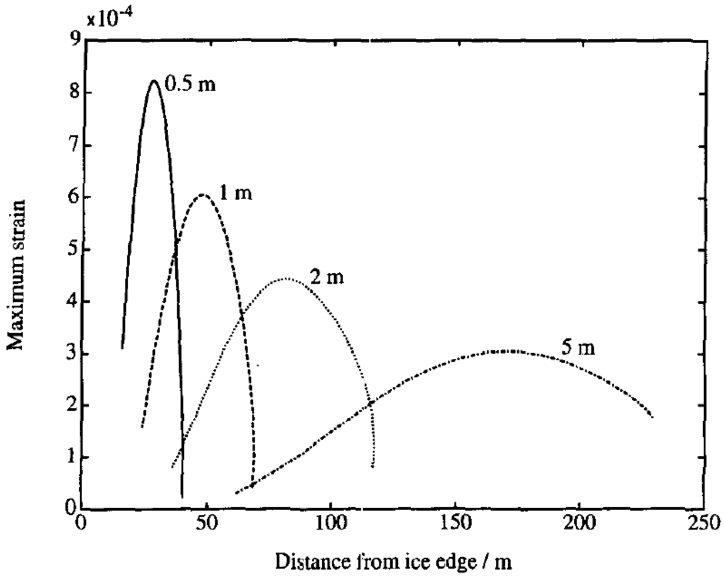

Figure 2.12 Plot of maximum strain vs. penetration at ice thicknesses of 0.5 m, 1 m, 2 m, and 5 m. The greatest strain attained for 0.5 m ice is at a penetration of 27.7 m by a 5.5 s period wave, 47.9 m for 7.25 s in 1 m of ice, 81.5 m or 9.5 s in 2 m of ice, and 170.9 m for 14.5 s in 5 m of ice [*Squire*, 1983; *Squire, et al.*, 1995]

Figure 2.13 Ice floes after the ice has been broken up by an incoming wave field. Note the very regular ice floes that are broken up by the waves, this can be explained by the distance of the maximum strain induced by the waves, see fig 2.11. From www.wikiwaves (Picture taken by V.A. Squire [*Squire*, 1984].

2.3.2 Scattering model in transport form

To describe the wave energy is it convenient to introduce the intensity function *I* describing the rate of flow of energy traveling in any given direction, and to describe how this function depends on the material that the waves travel through [*Dixon and Squire*, 2000; 2001; *Masson and LeBlond*, 1989; *Meylan and Masson*, 2006; *Meylan, et al.*, 1997; *Perrie and Hu*, 1996]. This formalism is based on a wide literature on waves in scattering media. If scatter and dissipation are included the general equation for energy propagation through a scattering medium is

$$\frac{1}{c_g}\frac{\partial}{\partial t}I(\mathbf{r},t,\theta)+\hat{\theta}\cdot\nabla I(\mathbf{r},t,\theta)=-\beta(\mathbf{r},\theta)I(\mathbf{r},t,\theta)+\int_0^{2\pi}S(\mathbf{r},\theta,\theta')I(\mathbf{r},t,\theta')d\theta' \qquad (2.20)$$

where $\mathbf{r}=(x,y)$, θ is the angular direction, $\beta(\mathbf{r},\theta)$ is the dissipation of energy and $S(\mathbf{r},\theta,\theta')$ is the scattering kernel reflecting the physical situation. The major problem is to find the appropriate scattering kernel, and this is a challenge for the case with wave ice interaction. Masson and LeBlond [1989] outlined a scattering kernel based on theories describing the movement of solid circular ice objects in a wave field. However, an extension of the problem to flexural ice objects has recently been provided [*Meylan and Masson*, 2006]. For the wave-ice system the equations can be written

$$\frac{1}{c_g}\frac{\partial}{\partial t}I+\hat{\theta}\cdot\nabla I=\int_0^{2\pi}\frac{fi}{A_f}\left|D(\theta-\theta')\right|^2 I(\theta')d\theta'-\left(\int_0^{2\pi}\frac{fi}{A_f}\left|D(\theta-\theta')\right|^2 d\theta'+\sigma_a\frac{fi}{A_f}\right)I(\theta') \qquad (2.21)$$

where *fi* is the fraction of the area covered by ice, A_f is the average area of the floe, τ_a is the absorption cross section area. $D(\theta-\theta')$ is the scattering amplitude, and determining $D(\theta-\theta')$ is the main challenge in this framework [*Masson and LeBlond*, 1989; *Meylan and Masson*, 2006; *Meylan and Squire*, 1996; *Perrie and Hu*, 1996]. Further description on the scattering kernel and the application of this model is given in Sec. 3.2. Another relevant study is outlined by Dixon and Squire [2000, 2001] who used the *Bethe-Salpeter* equation to describe the transport of energy in an elastic plate with random material properties. The scattering model will be described in some more detail in Section 3.2.

2.3.3 Waves in viscous fluid

In the previous sections, the ice floes are considered to be solid. However, a different path may be taken where it is assumed that all ice particles are embedded into the ice such that the ice is considered to be a highly buoyant and viscous fluid that floats on top on ordinary water. The physical background is that the presence of strong wave motions implies that small ice particles (order mm) cannot attach to each other such that larger pieces of ice cannot be formed [*Martin and Kauffman*, 1981]. Under freezing conditions with intense wave action very viscous and buoyant ice slush is created. Another physical situation of great interest is the pancake ice field characterized by many small (order 0.1-1m but with cakes up to 10 m size) ice floes surrounded by slush ice. Furthermore, parts of the MIZ may under certain circumstances be characterized by very small ice floes and with a high density of ice pieces that can be described as “slush ice” [*Frankenstein, et al.*, 2001][3].

To describe this situation, a model based on a two layer system can be outlined to describe the most pertain parts of the slush ice system [*De Carolis and Desiderio*, 2002; *Weber*, 1987]: the model is further described in Appendix C. The upper layer consists of viscous and buoyant slush ice while the lower layer consists of water. A schematic picture of the physical scenario that we consider is presented in Fig. (2.14). This type of system was initially described by Weber [1987] using a Lagrangian approach; however, as this mathematical framework is not common to most geophysical fluid scientists we outline a similar model based on the more standard Eulerian framework in Appendix C.

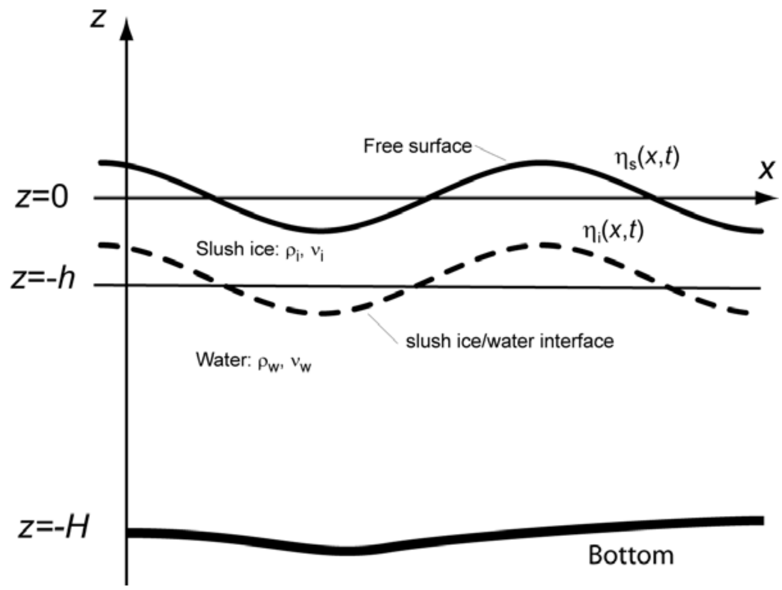

Figure 2.14 Schematic picture of the model for slush ice.

2.3.3.1 Outlining the viscous two-layer model

The basis of the model is the linear Navier-Stokes equations where friction is accounted for;

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2},$$
$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\frac{\partial^2 w}{\partial x^2} - g, \tag{2.22a-c}$$
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

The velocities are described in terms of the velocity potential, ϕ, and the stream function, ψ,

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial y}, w = \frac{\partial \phi}{\partial y} + \frac{\partial \psi}{\partial x}, \tag{2.23a, b}$$

[3] Notably, slush ice is used for a water-ice mixture of very small ice crystals; water-ice mixture with larger ice pieces of order cm to m appears not to have any specific name to the authors knowledge

The governing equations can thus be written [*Lamb*, 1932]

$$\nabla^2 \phi = 0,$$
$$\frac{\partial \phi}{\partial t} + \frac{P}{\rho} - \varphi = 0, \qquad (2.24a\text{-}c)$$
$$\frac{\partial \boldsymbol{\psi}}{\partial t} + \upsilon \nabla^2 \boldsymbol{\psi} = 0,$$

where φ is the potential arising from the gravity. The solutions can be written as [*De Carolis and Desiderio*, 2002; *Lamb*, 1932]

$$\phi = \left(Ae^{kz} + Be^{-kz}\right)e^{i(kx-\omega t)},$$
$$\psi = \left(Ce^{\alpha z} + De^{-\alpha z}\right)e^{i(kx-\omega t)}, \qquad (2.25a, b)$$

where the dispersion relation reads (applying 2.22c)

$$\alpha^2 = k^2 - i\frac{\omega}{\upsilon}. \qquad (2.26)$$

The solutions in 2.25 should be applied at both layers and there is thus eight constant that have to be determined using eight boundary conditions (see appendix C). The constants has not yet been determined analytically and some type of numerical solutions needs to be used [*De Carolis and Desiderio*, 2002]: however, it should be noted that an approximate analytical expression has been found using a Lagrangian approach to the problem [*Weber*, 1987].

The two layer slush ice model has been used to explain the wave attenuation in both laboratory experiment [*Newyear and Martin*, 1997], see Fig. (2.15), and wave attenuation in pancake ice (Fig. 2.16) [*Wadhams, et al.*, 1988]. Furthermore, there are evidence that the viscous model describe wave dynamics (i.e., it can explain observations on the dispersion relation) in pancake ice or in ice conditions with scattered ice conditions [*Wadhams, et al.*, 2002]. It should be noted that the model can give accurate description of wave attenuation rates and on predicting the dispersion relation for not-so-solid ice; however, very different values of the viscosity of the ice slush layer has to applied for each application reducing the usefulness of the model for practical and prognostic applications. Dimensional analysis have been used to derive a model for estimating the viscosity of the slush layer [*Liu, et al.*, 1991] but no thorough framework has been presented to the authors knowledge.

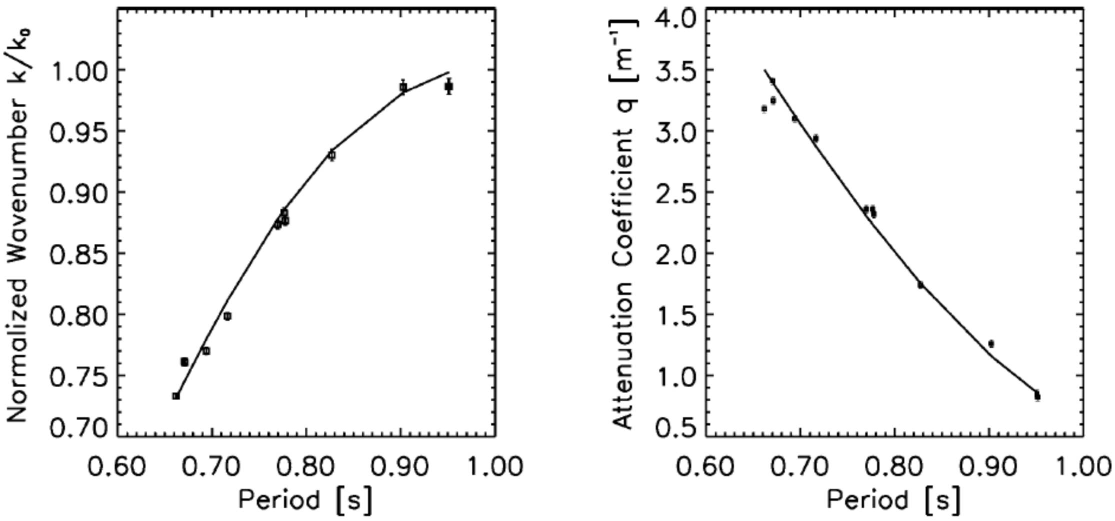

Figure 2.15 Comparison between model results and laboratory experiment carried out with waves in slush ice [*Newyear and Martin*, 1997]. Left figure shows the dispersion relation and the right figure displays the attenuation coefficient. The viscosity of the upper layer was used to fit data and left figure uses ν_i=2.94 10^{-2} m^2 s^{-1} while the right panel uses ν_i=3.68 10^{-2} m^2 s^{-1}, in lower layer ν_i=1.8 10^{-6} m^2 s^{-1} were used [*De Carolis and Desiderio*, 2002].

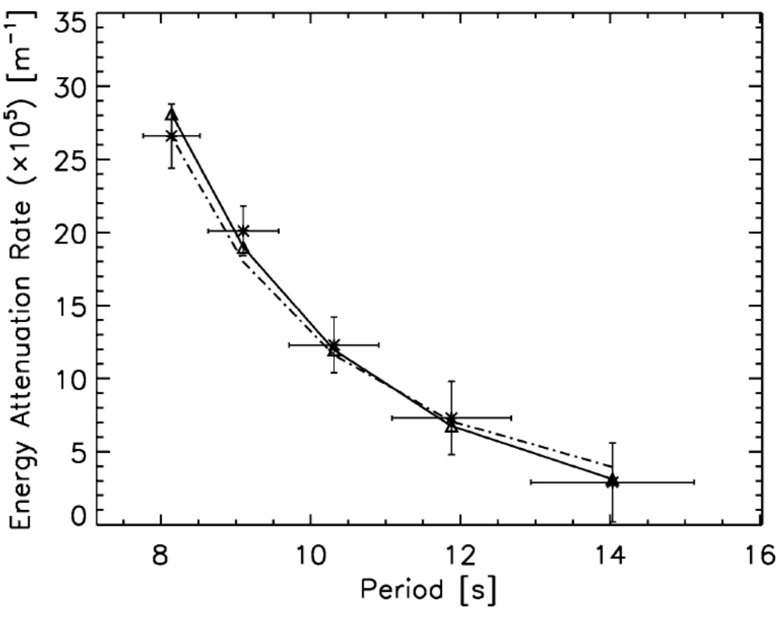

Figure 2.16 Comparison of data from Greenland 4 September 1979 [*Wadhams, et al.*, 1988] and curves with the Weber model (dash dot line) and the viscous two-layer model presented here. The viscosity was used to fit model with data and ν_i=1.95 10^{-3} m^2 s^{-1} was used for the present model while Weber used ν_i=1.95 10^{-3} m^2 s^{-1} [*De Carolis and Desiderio*, 2002].

2.4 Miscellaneous on waves in ice

2.4.1 Estimating ice thickness from wave observation

The ice thickness in itself is a parameter that is difficult to measure; ice thickness cannot be observed directly by remote sensing and the high irregularity in ice thickness makes it laborious to measure by taking discrete samples e.g., using ship or helicopter [*Wadhams, et al.*, 1987]. A relatively clear picture of the ice thickness can be obtained using submarines [*Wadhams*, 1981; 2000] or underwater vehicles such as Automatic Underwater Vehicle (AUV) [*Hayes, et al.*, 2007; *Wadhams, et al.*, 2006] or Remotely Operated underwater Vehicle (ROV). However, these methods require expensive equipment and are costly in the long run.

It is unfortunate that sea ice thickness is very difficult to measure as it is one of the major parameters for planning shipping or other offshore activities in ice covered areas. However, if accurate measurements of wave characteristics in ice, such as wave length and wave period, could be performed with remote sensing, for instance by satellites or aircraft, and the ice-wave interaction would be known to sufficient degree, this types of measurements could be used to find the ice thickness over large areas [*Nagurny, et al.*, 1994]. The idea is to observe certain wave properties in the ice using some kind of remote sensing, if the relation between the wave properties and the ice thickness is known these observations on the wave properties can be used to estimate the ice thickness. Schematically we can write the dispersion relation for waves in ice as

$$\omega(k,h) = 0$$

where ω is its angular frequency, k is the wave number, and h is the ice thickness. If we can make observations on ω and k (e.g., by satellite observation), it will become possible to calculate h if the above relation is well known. The thickness of a pancake ice field was has been estimated with some success [*Wadhams, et al.*, 2002; *Wadhams, et al.*, 2004]; however, there are still some questions on the reliability of these estimates. The major uncertainty is most likely the fundamental uncertainty how wave propagate through different ice fields such as, slush ice, pancake ice, broken ice, and solid ice.

2.4.2 High amplitude waves in solid ice: RV Polarstern in Weddell Sea 1986

The present description is based on Liu and Mollo-Christensen [1988]. In 1986 the research vessel RV Polarstern were steaming through sea ice with thickness of roughly 0.8 m and they were about 560 km from the ice and open-water border. Suddenly waves with period 18 s (and estimated length of 250 m) and a wave amplitude of about 1 m appeared. At the same time as these waves appeared, the ice and the sea became more vibrant with significant rafting taking place during a short period of time, and there were numerous new ice ridges with thicknesses up to 2 m. Furthermore, within a few hours the original continuous ice cover were broken up in small floes, with the majority of the floes being smaller than 50 m. With three machines running, RV Polarstern had some difficulties of maneuvering through the new ice conditions. The rapid appearance of a wave field and the associated new ice field were not foreseen and represented a surprising and uncomfortable situation.

A note of some interest here is that the estimated wave lengths were significantly smaller as compared to waves of similar frequencies in open water. Apparently, the presence of stiff sea ice altered the wave length significantly. Another interesting observation is that after the wave broke the ice into small floes such the stiffness of the ice were broken, the wave length increased toward the open ocean value. Thus, it may be concluded that even though RV Polarstern was way inside the pack ice, which is an area characterized by calm wave and ice conditions, the ice cover itself amplified the small amplitude waves that reached the position of RV Polarstern. It may be concluded that the ice thickness were of correct magnitude to slow down the propagation of wave energy such that high wave amplitudes are created locally from low amplitude waves moving into the area [*Liu and Mollo-Christensen*, 1988][4]. Furthermore, it can be shown that that the non-linear wave-wave interaction described by the non-linear Schrödinger equation can become very rapid under an ice cover such that the wave amplitude of wave packages can increase very rapidly [*Liu and Mollo-Christensen*, 1988].

The sudden appearance of the high energetic waves deep inside a pack ice has not been observed scientifically before or after this incident (to the authors knowledge). The rareness of these events implies that they are difficult to study, and it is difficult to make a solid statement of what caused the high amplitude waves and the violent ice cracking that was observed. Furthermore, it may be concluded that the described phenomena is rare; however, the rareness of such phenomena cannot be estimated representing one uncertainty in operating in ice covered areas in vicinity (say 500 km) of open water.

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[4] Here it should be noted that the transport of wave energy depends on the wave energy itself, i.e., the square of the wave amplitude, and the group velocity of the wave. When the group velocity becomes small the wave amplitude must increase to keep the wave energy transport constant, see Appendix A.

3 Present status of modeling waves in sea ice

3.1 Review of model systems

3.1.1 Wave models

It is common scientific knowledge that waves are generated by the wind, and that they dissipate by friction or wave breaking [*Cavalieri*, 2007; *Komen, et al.*, 1994; *Phillips*, 1977]. Furthermore, different wave lengths are affected differently by wind forcing and dissipation such that there is a spectrum of wave lengths (or wave frequencies) that evolves due to wind action and dissipation. It is also well known that waves with different wave length interact and there is a continuous exchange of energy between waves of different wavelengths. Thus some waves grow while other decline, and in such manner that the total energy of the wave spectrum is conserved. There are different types of non-linear wave-wave interactions but the most important interaction involves four different wavelengths[5]. The net outcome of the interaction is that long waves gain energy while shorter waves loose energy. Furthermore, the non-linear wave-wave interaction implies that the wave spectrum tends to a well-defined shape under homogenous conditions [*Komen, et al.*, 1994; *Komen, et al.*, 1984; *Lavrenov*, 2003; *Phillips*, 1977].

The frequent characterization of wave models into first, second, and third generation wave models are based on how the models treat the non-linear wave-wave interactions.

- First generation models do not describe the non-linear wave-wave interactions.
- Second generation models use a highly parameterized estimate of the non-linear wave-wave interactions.
- Third generation models calculates the non-linear wave-wave interactions according to first principles, although some simplifications are made to speed up the numerical calculations. The most well-known third generation wave models are WAM [*Hasselmann, et al.*, 1988; *Komen, et al.*, 1994], SWAN [*Booij, et al.*, 1999] WaveWatch [*Tolman*, 1991; 2002], TOMAWAK [*Benoit, et al.*, 1996], and CREST [*Ardhuin, et al.*, 2001]. *At met.no WAM and SWAN are used.*

The main quantity we want to know from a wave model is the energy, E, as a function of wave frequency, f, and the wave direction, φ, i.e., we want to know $E(f,\varphi)$. However, for modeling purposes it turns out that wave action density $A=E/f$ is a more appropriate quantity to model as its evolution can be described in a simple way. (The energy E is not conserved if the frequency varies due to e.g., changes in the current velocity; however E and f varies in such way the E/f is conserved).

3.1.2 Prognostic equation for action density (i.e., wave spectra)

The prognostic equation for the wave action density for open water is

$$\frac{\partial A}{\partial t} + \left(U + c_g\right)\cdot \nabla_h A = S_{in} + S_{nl} + S_{diss} \qquad (3.1)$$

[5] Note that this type of wave-wave interaction does not describe the rapid development of so called freak waves; freak waves are generally modeled through the non-linear Schrödinger equation

where

- U, c_g are ocean current speed and the group velocity, respectively.
- S_{in} is the input of energy, essentially from wind.
- S_{nl} represents the non-linear wave-wave interactions.
- S_{diss} is the dissipation of wave energy by wave breaking, and frictional and turbulent forces.

The non-linear wave-wave interaction is considered to be well-known today albeit there is an intense research on finding more efficient numerical methods to evaluate the expensive calculations. The wave input and the dissipation are both based on ad hoc parameterizations: the input of waves is considered as relatively well known while there are more uncertainties regarding the wave dissipation (e.g., wave breaking is an important parameter that is difficult to study and parameterize). In any way, wave input and wave dissipation are areas of active research. However, wave models have been thoroughly validated and there is no reason to question the order of magnitude of the parameterizations [*Cavalieri*, 2007].

3.2 Wave modeling in ice covered areas

In ice conditions the formulation becomes more uncertain but it seems reasonable to consider the following prognostic equation for wave action density [*Masson and LeBlond*, 1989; *Perrie and Hu*, 1996]

$$\frac{\partial A}{\partial t}+\left[\left(U+c_g\right)(1-A_i)+\left(U_i+c_{g-i}\right)A_i\right]\cdot\nabla_h A=\left(S_{in}+S_{nl}+S_{diss}\right)(1-A_i)+S_{nl}^{i}A_i+A_iS_{ice} \tag{3.2}$$

where (in addition to the above equation)

- A_i is the fraction of the area covered by ice
- S_{nl}^{i} is the non-linear wave-wave interaction under ice cover [*Polnikov and Lavrenov*, 2007]. It may be noted that using a "standard" wave-wave interaction model will not give severely wrong results; nevertheless, there are certain affects of the ice cover.
- S_{ice} is the scattering and dissipation from ice floes, ice ridges etc and includes dissipation under ice conditions.

There have not been many attempts to run wave models using the above configuration. However, one study exists [*Perrie and Hu*, 1996] and some description of this model has been reproduced here: We do not intend to give a full description of the model, rather we want to give an impression of the complexity of the model. The model is based on an extension of the energy scattering model described by Masson and LeBlond [1989]; however it should be recognized that this model is based on the movement of solid objects such that features depending on the flexural properties of the ice are not described.

It have been suggested [*Masson and LeBlond*, 1989; *Perrie and Hu*, 1996] that the ice term can be written in terms of a transformation tensor T_{ft}^{ij} such that

$$S_{ice}=E(f,\varphi)T_{fl}^{ij}, \tag{3.3}$$

where

$$T_{fl}^{ij} = A^2 \left[\beta \left| D(\varphi_{ij}) \right|^2 \Delta\varphi + \delta(\varphi_{ij}) \left(1 + \left| \alpha_c D(0) \right|^2 \right) + \delta(\pi - \varphi_{ij}) \left| \alpha_c D(\pi) \right|^2 \right]. \tag{3.4}$$

Here δ is the Dirac delta function and $\Delta\varphi$ is the angular increment in φ_i and $\varphi_{ij} = \left| \varphi_i - \varphi_j \right|$.
The parameter α_c is the coherent scattering coefficient and depends on the distance between ice floe centers D_{av} and floe radius a and is described by

$$\alpha_c = \frac{2\pi}{k} e^{i\pi/4} \frac{2}{\sqrt{3} D_{av}^2 \sqrt{1 - 8a^2 / \sqrt{3} D_{av}^2}} \int_0^\infty e^{ikx_s} \left(1 - \frac{8a^2}{\sqrt{3} D_{av}^2} \right)^{x_s/2a} dx_s . \tag{3.5}$$

The coefficient β is a cumulative expression for the effective density of wave scatters, $\rho_e(r)$, and is defined as

$$\beta = \lim_{r \to \infty} \int_0^R \rho_e(r) dr . \tag{3.6}$$

If the scattering process is limited to some time period Δt it follows

$$\beta = \int_0^{R_{\max}} \rho_e(r) dr , \tag{3.7}$$

where R_{max} is the maximum distance travelled by waves during time period Δt. The effective density of ice floe scatters, $\rho_e(r)$, radiating waves to position **r**, is

$$\rho_e(r) = \frac{fi}{\pi a^2 \sqrt{1 - 4fi/\pi}} \left(1 - \frac{4fi}{\pi} \right)^{r/2a} , \tag{3.8}$$

where fi represents ice cover concentration. The coefficient A is defined as

$$A = \left(1 + \left| \alpha_c D(0) \right|^2 + \left| \alpha_c D(\pi) \right|^2 + \beta \int_0^{2\pi} \left| D(\varphi) \right|^2 d\varphi + f_d \right)^{-1/2} , \tag{3.9}$$

where

$$\begin{aligned} f_d &= \left(e^{\rho_0 \sigma_a c_g \Delta t} - 1 \right), \\ \rho_0 &= 2 / \sqrt{3} D_{av}^2 , \end{aligned} \tag{3.10}$$

for a rigid ice floe array [*Masson and LeBlond*, 1989].
The cross section scattering coefficient $D(\varphi_{ij})$ is given in terms of heave, $D_1(\varphi_{ij})$, surge, $D_2(\varphi_{ij})$, pitch, $D_3(\varphi_{ij})$, and diffraction, $D_4(\varphi_{ij})$, such that

$$D(\varphi_{ij}) = D_1(\varphi_{ij}) + D_2(\varphi_{ij}) + D_3(\varphi_{ij}) + D_4(\varphi_{ij}) . \tag{3.11}$$

The components are formulated such that

$$\begin{aligned} D_b(\varphi_{ij}) = &-\frac{\omega}{k} \frac{\xi_b}{H/2} \sqrt{\frac{2\pi}{k}} C_0 \cosh(kh) e^{-i\pi/4} \\ &\times \sum_{j=1}^{N} \sum_{l=0}^{\infty} f_{bl}(s_j) \cosh\left[k(Z_j + h) \right] J_l(kR_j) e^{-i\pi l/2} R_j L_j \cos(l\varphi_{ij}), \end{aligned} \tag{3.12}$$

where $H/2$ is the wave amplitude, h is the water depth, J_l is the Bessel function of the first kind of order l. Specifications of the body motion amplitudes ξ_b, source strength function f_{bl}, and the geometrical elements R_j, Z_j, L_j, s_j, and C_0 can be found in Perrie and Hu [1996]. The scattering model will give predictions on the attenuation coefficient at all grid points at every time step. An example of the attenuation spectrum is given in Fig. (3.1). In addition to the wave model, the movements of the ice floes have to be considered [*Perrie and Hu*, 1997], this is not described here but may be a crucial component of a comprehensive wave-ice model. It should be noted that there are also other models based on relatively similar principles but these have not been included into a geophysical wave model [*Dixon and Squire*, 2000; 2001; *Meylan and Masson*, 2006; *Meylan, et al.*, 1997].

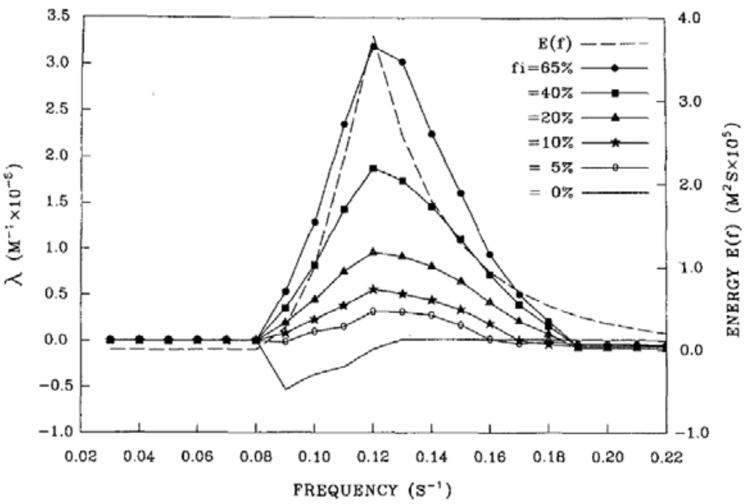


Figure 3.1 The wave attenuation λ as a function of frequency f and ice cover concentration fi for the model by Perrie and Hu (1996). The energy impinging in MIZ is given by E(f) assuming a 20-m floe diameter and a 1.5 m floe thickness [*Perrie and Hu*, 1996].

3.3 Operational systems

Today met.no runs several operational models that would be useful for modeling of waves in ice. There are a number of models that can be used and the exact choice of models may depend upon the areas where the wave-in-ice model should be applied. All forecast models at met.no runs on several resolution and different areas: here we take the *Barents Sea* area as an example; for other areas a different choices may be made. Each description outlines the main features of the model, the model domain, and a forecast from the model made on Mars 29, 2008.

3.3.1 Meteorological models

The meteorological forcing is important for forcing the ocean currents, the wave model and the ice-floe ice-margin movements. The main meteorological model at met.no is the High Resolution Lower Atmosphere Model (HIRLAM). Besides the output from the HIRLAM model, met.no daily receives global forecast from the European Centre for Medium Range Weather Forecast (ECMWF).

Specifications on HIRLAM10.

Parameters: Wind, temperature, pressure, humidity, cloud liquid water, precipitation, long and short wave radiation. Other parameters on request.

Area coverage: Scandinavia, Norwegian Sea, Barents Sea and part of the North East Atlantic.

Spatial resolution: Approximately 10 km.

Vertical resolution: The model has 40 levels in the vertical. Surface data and data on specified pressure or height levels available on request.

Availability: 4 runs a day available in suites 00, 06, 12, 18 UTC. Available approximately at 02:40, 10:40, 14:40 and 22:40 UTC.

Time steps: 1-hourly from T+0 to T+60.

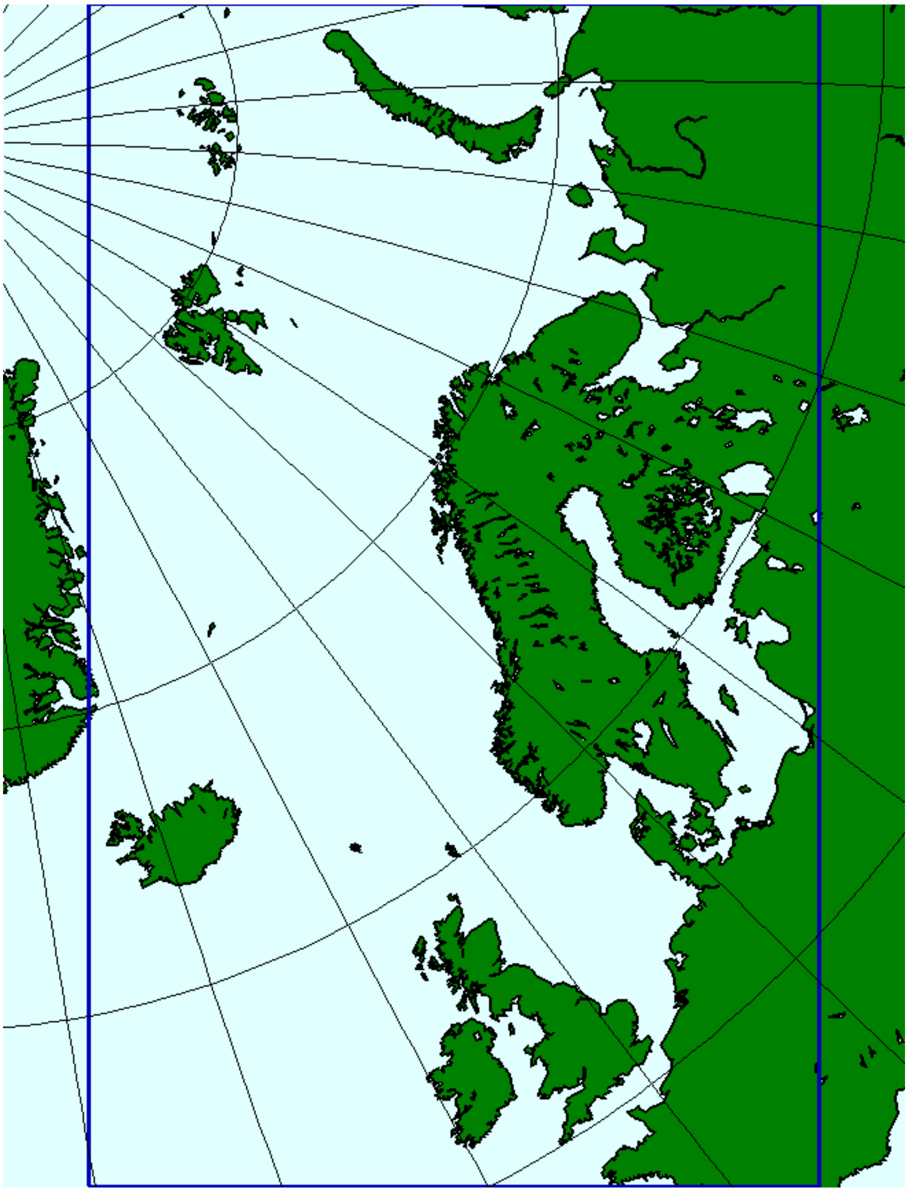


Figure 3.2 Model domain for HIRLAM10 (within blue frame)

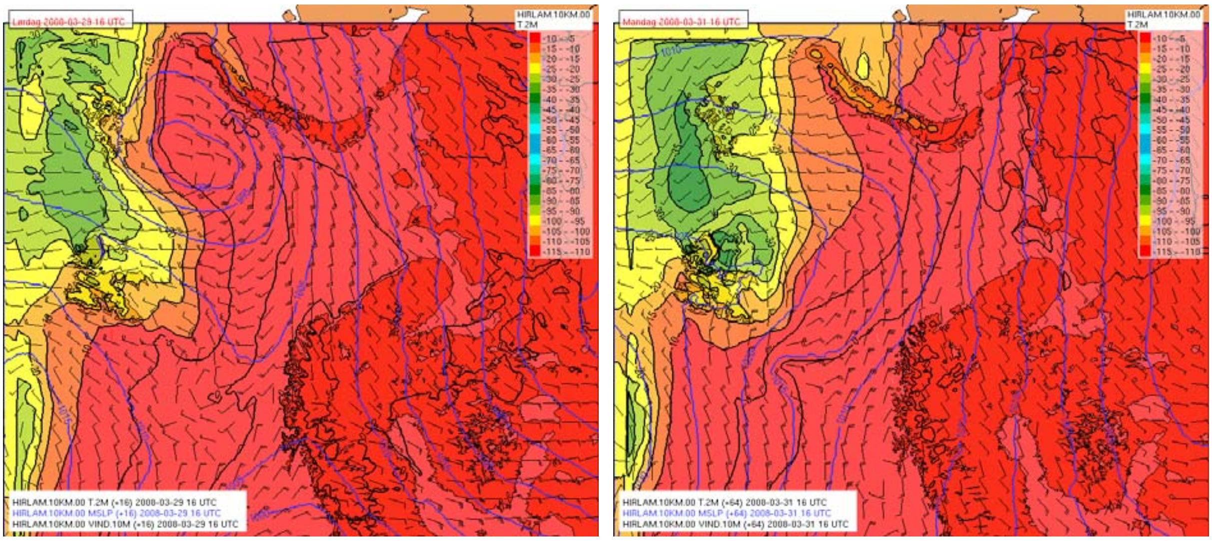


Figure 3.3 Results from the HIRLAM 10 model for Mars 29, 2008. Left panel is the forecast for 16.00 on Mars 29 and the right panel is the forecast for Mars 31 at 16.00. The figure shows the wind vector, the temperature at 2 m (color) and mean sea level pressure.

3.3.2 Oceanographic models (including ice models)

The oceanographic model is important for determining the movements of the ice, which is considered to be a part of the ocean model. Today met.no uses the MI-POM model (which is a development of the Princeton Ocean Model (POM)) as the prognostic ocean model but it has been decided to change to the Regional Ocean Model (ROMS) model within the next few years. The sea ice model has only been implemented in the Arctic Ocean model at the present stage; thus, if a dynamic ice model is required this will have some implications on the choice of model or the inclusion of the ice model into other operational setups.

Specifications on Arctic-20km:

Parameters: Sea surface elevation, Currents, Salinity, (Potential) Temperature, Ice concentration, Ice thickness, Ice velocities. Other parameters possible on request.

Area coverage: Arctic Ocean, Norwegian Sea, Barents Sea, North Sea. Approximate coverage in degrees: Circumpolar north of 50N in the Atlantic.

Spatial resolution: Approximately 20 km.

Vertical resolution: Depths: 0, 3, 10, 25, 50, 75, 100, 500, 1000 m.

Availability: 1 run a day available in 00 UTC. Results available approximately at 03:00 UTC.

Time steps: 6-hourly from T-30 to T+168.

Figur 3.4 Model domain for Arctic-20km (green)

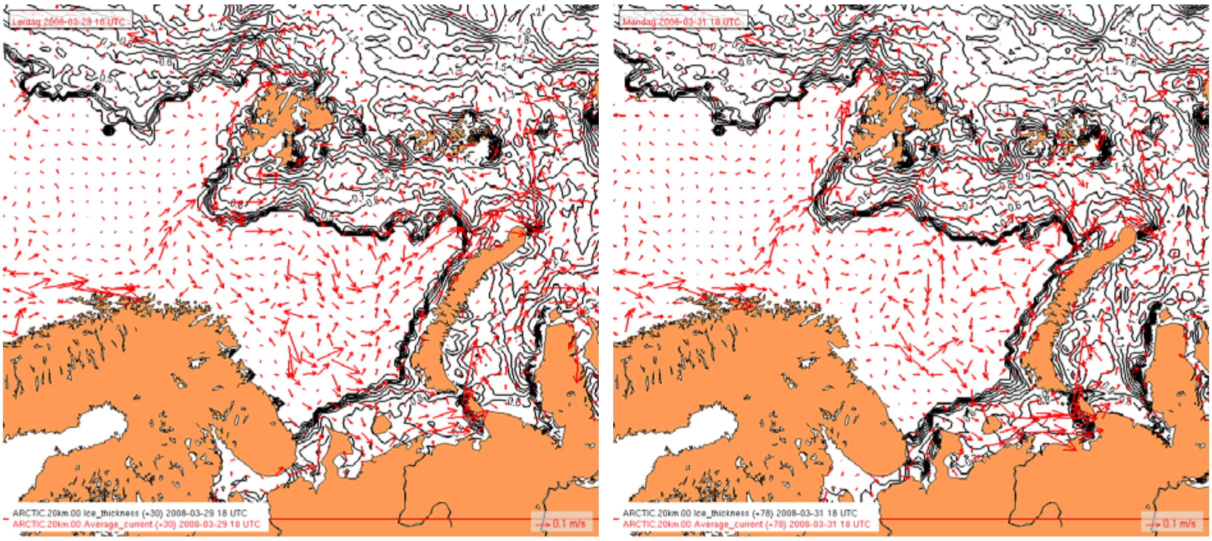


Figure 3.5 Results from the ARCTIC 20 model for Mars 29, 2008. Left panel is the forecast for 18.00 on Mars 29 and the right panel is the forecast for Mars 31 at 18.00. The figure shows the current vector and the ice thickness (contour).

Specifications on Nordic-4km:

Parameters: Sea surface elevation, Currents, Salinity, (Potential) Temperature. Other parameters possible on request.

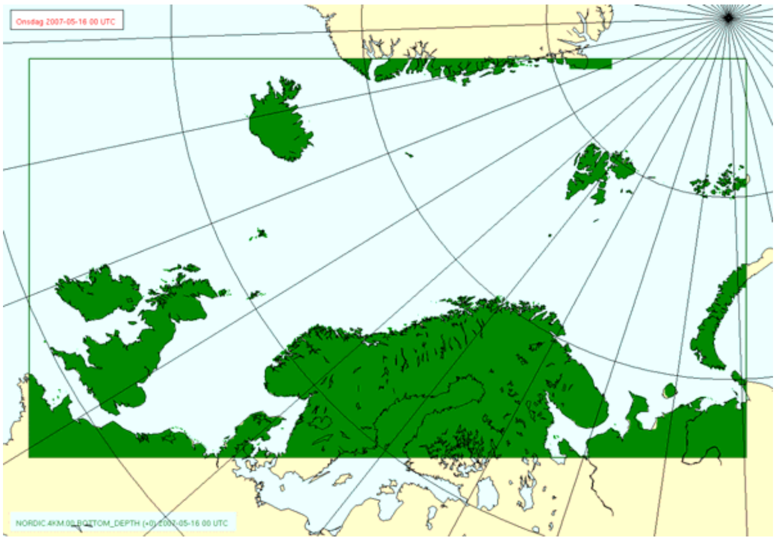
Area coverage: Norwegian Sea, Barents Sea, North Sea. Approximate coverage in degrees N, S, W, E: 85, 50, -25, 60.

Spatial resolution: Approximately 4 km.

Vertical resolution: Depths: 0m. Other depths available on request for horizontal subsection of the total domain.

Availability: 2 runs a day available in 00 and 12 UTC. Results available approximately at 03:00 and 15:00 UTC.

Time steps: 1-hourly from T-18 to T+60. 2-hourly for other depths than 0m.



Figur 3.6 Model domain for Nordic-4km (green)

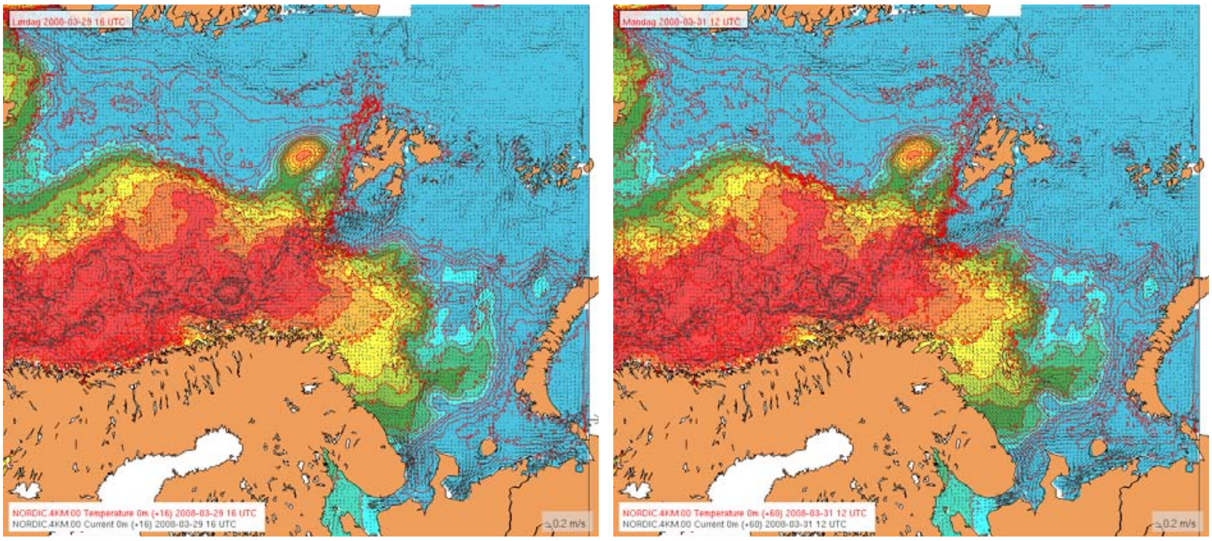


Figure 3.7 Results from the Nordic 4 model for Mars 29, 2008. Left panel is the forecast for 16.00 on Mars 29 and the right panel is the forecast for Mars 31 at 12.00. The figure shows the current vector and sea surface temperature (color)).

3.3.3 Wave models

The wave model is of course at the heart of a forecast model for waves-in-ice. The main forecast model at met.no is the Wave Analysis Model (WAM), but some regional high resolution wave forecasts are made with the Simulating WAves Nearshore (SWAN) model.

Specifications on WAM.10km (nested in WAM.50km)

Parameters: Significant wave height, mean period, peak period, peak direction. Wind sea: significant wave height, peak period, peak direction. Swell: significant wave height, peak period, peak direction.

Area coverage: North Sea, Norwegian Sea, Barents Sea. Approximate coverage in degrees N, S, W, E: 84, 54, -25, 65.

Spatial resolution: Approximately 10 km.

Vertical resolution: Only sea surface fields.

Availability: 2 runs a day available in 00 and 12 UTC. Results available approximately at 03:00 and 15:00 UTC.

Time steps: 1-hourly from T-11 to T+60.

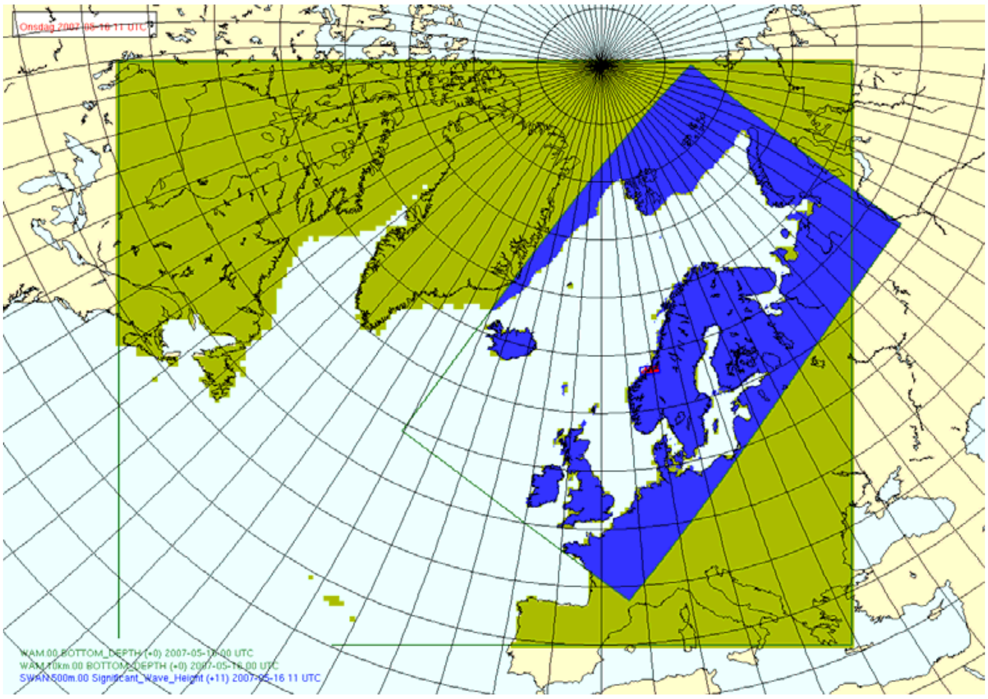


Figure 3.8 Model domains for WAM.50km (green) and WAM.10km (blue)

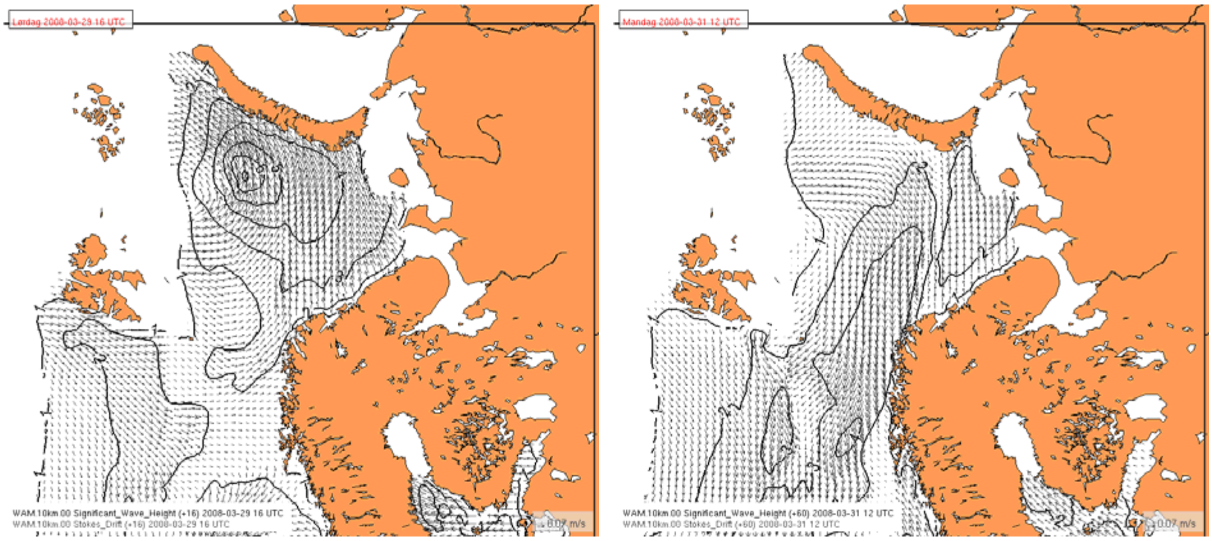


Figure 3.9 Results from the WAM 10 model for Mars 29, 2008. Left panel is the forecast for 16.00 on Mars 29 and the right panel is the forecast for Mars 31 at 12.00. The figure shows the Stokes drift current vector (a measure the transport of wave momentum transport) and significant wave height (contour). Observations on ice field are used to mask the wave model.

4 Future possibilities for modeling and forecasting of wave propagation into sea ice

The complexity of the basic physical problem, and the mathematical complexity of the state-of-the-art models describing wave and ice interaction, will also be reflected in any model describing the coupled wave-ice system. Also the ice coupling to atmospheric forcing and ocean currents needs to be described. Such coupled models are certainly well beyond the present state-of-the-art in modeling. Having said this, it should equally well be acknowledged that simpler model systems can be utilized that give added value compared to the present day knowledge.

4.1 Overview of processes to be included

To make accurate predictions of the wave field in an ice covered sea several aspects must to be resolved. The most important aspects we need to consider are: the structure of the ice cover, the wave-ice interaction, and the size of the zone we are interested in. All these aspects must be considered to provide accurate description of the wave ice interactions.

4.1.1 The structure of the ice cover

The structure of the ice cover is essential for the attenuation of wave energy in ice fields. Here it may be noted that much of the uncertainties about interpreting field experiments lies in the absence of data on the ice conditions. Given that the waves and ice is such a coupled system it may be difficult to separate the wave and the ice fields in a model system for predicting waves-in-ice. The following must be known for accurate prediction of wave ice interaction.

- Ice thickness.
- Floe concentration and its size distribution.
- Ice type needs to be described (e.g., what is the concentration of small ice particles, to what extent is the ice characterized as slush ice etc).

These quantities are not readily observed to the degree required in a forecasting model. Furthermore, the ice field changes very rapidly in both space and time such that it may be difficult to base a wave forecasting system on observations of ice properties.

It is likely that a forecasting system for waves-in-ice must be complemented with a detailed ice forecasting system describing the ice floe distribution and the degree of slush ice. Notably, this is not considered in any of the existing ice models of today. For instance, waves will have a strong influence on the floe distribution through the efficiency of the waves to break ice floes into pieces. *Thus we conclude that the prediction on ice characteristics is probably the weakest link in any attempt to build a forecasting system for waves-in-ice.*

4.1.2 Wave ice interaction

The wave ice interaction will lie at the heart of any model for predicting wave energetic in ice covered seas. When waves enter an ice field there is always some scattering and dissipation of wave energy. Observations show that the wave energy declines essentially exponentially from the first appearance of ice floes and further into the ice. How deep the waves will penetrate depends on the ice characteristics but it is reasonable to assume that waves penetrate at least 2-5 km in areas with high concentration of large ice floes, to distances of hundreds of km in pancake ice or solid ice.

The attenuation of waves is the results of a complex interplay between waves and ice, involving both scattering and dissipative forces. The wave-ice system is by nature a very complex system involving interaction between strong fluid motion and floating (and flexible) objects (i.e., ice floes). These ice floes will in its turn affect the waves creating a highly coupled dynamical system. The characteristics of the ice cover are highly variable and changes quickly with location and time, the wave-ice interaction accordingly depends on the local features of the ice cover. Today there is no general framework that accounts for ice and wave interaction for a wide range of ice and wave constellations. Some strength and shortcomings of present day knowledge are outlined below.

Scattering processes needs to be described. There exist advanced theories that describes the interaction between waves and solid ice particles [*Masson and LeBlond*, 1989; *Perrie and Hu*, 1997] or flexible ice covers [*Dixon and Squire*, 2001; *Fox and Squire*, 1994; *Kohout and Meylan*, 2008a; *Kohout and Meylan*, 2008b; *Kohout, et al.*, 2007; *Meylan*, 2002; *Meylan and Masson*, 2006; *Meylan and Squire*, 1994; 1996; *Meylan, et al.*, 1997; *Squire*, 1995; 2007; *Squire and Williams*, 2008; *Vaughan, et al.*, 2007; *Williams and Squire*, 2006][6]. Many of these theories may be evaluated for practical applications, but this is probably not a straightforward task. Several models are of course validated against the sparse data sets that exist; however, the models also need to be evaluated for robustness and for computational costs. The difficulty to obtain mathematical solutions to even relatively "simple" physical problems has been a major roadblock for studying more practically oriented problems. Numerical methods would represent one way forward: however, the dynamical features that creates problem in obtaining analytical solution will also be an obstacle employing numerical methods[7]. We conclude that numerical methods are probably underutilized for studies of the wave-ice system.

Floe floe interaction must be evaluated. One important aspect of the wave ice interaction involves floe floe interactions, which are known to affect the damping of waves through scattering and dissipative forces. A thorough understanding of these interactions is most likely needed to create realistic models for waves-in-ice. This is especially true for the interior marginal ice zone that is characterized by a dense ice floe concentration. Despite the importance of floe floe interaction in a wavy ice field, there are few theoretical studies that address this problem.

Friction must be described in a realistic way. Frictional effects have been neglected to a large extent. Nevertheless, there are some models that have proven very valuable in a number of situations [*De Carolis and Desiderio*, 2002; *Wadhams, et al.*, 2002; *Weber*, 1987]. For instance, there exist a few studies that include the role of viscous forces on the wave motion in an ice field. Furthermore, frictional effects seem to be overlooked in studies of wave ice interaction, even though recent development focuses on ice cover with complex form. Furthermore, the scattering model conserves energy, and in the realistic case where externally

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[6] It should be noted that some of these theories use rather advanced mathematics and may be difficult to apply for non-mathematicians.

[7] For instance, the fourth order mixed derivative appearing in the boundary condition at the flexible ice cover is not easily handled by standard mathematical methods: One way around this problem is to formulate the boundary conditions using the Green function [*Lavrenov 2003, Lavernov and Novakov, 2000*]

forced waves continuously pump energy into the ice zone there must be some process to remove energy, and this can only be done by viscous processes.

4.1.3 Numerical resolution

In addition to these choices on the physical description of the system, some choice has to be made on how to resolve the wave-ice zone. One difficulty with making accurate predictions of the coupled wave ice field is the small and highly variable scales of the system. The wave-ice zone has a scale of order 1-50 km and it changes rapidly with changing forcing conditions. The most relevant operational models at met.no runs at resolutions of 10 km (atmospheric and wave models), and 4 km and 20 km for the ocean and ice model[8]. Thus the wave-ice zone would not be sufficiently resolved in today's models. Here it should be noted that the ice edge itself may induce a sea breeze that may be important for the evolution of the ice field. The sea breeze system is only marginally resolved with a 10 km grid posing a problem for accurate prediction of the ice field in sea breeze conditions.

The immediate question is if the resolution of the underlying model should be increased, or if a system that follows the ice edge should be developed, i.e., a system than only describes the wave-ice zone. In the latter case the position of the ice margin must be specified from model or observations, while the model predicts the characteristics of the wave-ice zone.

4.2 Evaluation of possibilities for including ice and wave parameters into the present operational wave and ice forecast model at met.no

A comprehensive wave-in-ice model will require

- An atmospheric model that provides atmospheric forcing fields.
- An ocean model that provides current fields and sea surface temperature.
- An ice model that describes the necessary ice characteristics.
- A wave model that includes wave ice interaction.

Furthermore, these models have to be run at a resolution that resolves the ice edge dynamics.

The wave ice interaction is not included in the present met.no models: otherwise, the basic models run operationally at met.no today. However, the models run at too low resolution for being optimal for wave-in-ice modeling. We thus conclude that the basic model systems needed for wave-in-ice predictions are up and running using state-of-the-art models although the ice model may need reworking. However, there is no wave-ice coupling in these models and one of the results from this study is that there is no single well accepted operational model, or easy applicable theoretical framework, for predicting the wave characteristics in an ice covered ocean. On the contrary, to the authors knowledge there exist no operational models for predicting waves in ice (personal contacts with European Centre for Medium Range Weather Forecast (ECMWF), Prof. Perrie at Bedford Institute in Canada, and Dr. Meylan at University of Auckland, New Zealand). We thus conclude that presently there is no knowledge on operational predictions on wave heights in sea ice.

During this literature study we have only encountered one realistic geophysical wave-ice model as of today [*Perrie and Hu*, 1996]; this model is not used operationally and employ a

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[8] The 4 km ocean model covers the main part of the Barents Sea but as of today it does not carry a dynamical ice model

scattering model that may be too simple. Furthermore, the ice characteristics must either be prescribed or modeled in any way [*Perrie and Hu*, 1997].

A few possibilities of increasing complexity is listed below

- Developing tables for the wave energy attenuation coefficients for different ice and wave conditions. This will imply that the user of such tables must have a basic understanding of the wave-ice system, and how it responds to changes in the forcing. Such lookup tables can be used both manually and within a numerical model, i.e., these lookup tables can be implemented in a wave model. Fairly accurate predictions on the wave-ice climate at certain conditions can most likely be made but more complex situations can probably not be captured. The creation of such tables will, however, require detailed understanding of relevant processes, field data, and output from modeling studies.
- Applying simple scattering, and wave dissipation, models using prescribed ice floe concentration and characteristics, i.e., the ice characteristics is determined by a table rather than being described prognostic.
- Designing a model that describes both the wave and ice conditions. This will be a rather challenging task even if a relatively simple scattering model may suffice.
- A wave-ice model based on the state-of-the-art knowledge of wave-in-ice theories and dynamics of ice floe and ice dynamics. The development of such models will undoubtedly contain both new laboratory experiments and field work for acquiring data for validating the model. The development of such model system will require a substantial effort.

Although the term simple models are used above, the work in setting up and developing these simple models should not be underestimated, wave-ice interaction is a very complicated field of science. Furthermore, any reliable forecasting system must be evaluated and validated routinely. Today, the data sets for validating wave-ice models are sparse and often there are significant gaps in the underlying data of the wave and ice fields such that it is not possible to evaluate the data set in a comprehensive way. Thus, new data sets may be needed to validate an operational wave-in-ice model.

We conclude that met.no has the capability to include some type of wave-in-ice model. However, the implementation of such model will require significant resources. Furthermore, new studies aiming at applying existing theories into scenarios relevant for practical purposes needs to be outlined for the development of a state-of-the-art forecasting system[9].

4.3 Suggestions for future studies

This limited literature review does not cover all the complex literature regarding wave-ice interaction, and the nearby field of wave and floating-object interaction. Nevertheless it seems clear from a practical application point of view that there are some mismatch between theoretical development, laboratory studies, and field experiments. Many of the most comprehensive field experiments are over 30 years old. New laboratory experiments that will shed new light on important processes and for verifying theoretical predictions are also needed.

[9] As there are no forecasting system as of today, all new forecast systems will essentially be beyond the state-of-the-art

The complexity of the waves-in-ice system probably implies that it has to be attacked on many fronts at the same time: theory, laboratory experiments, field experiments and numerical experiments must be considered simultaneous. Numerical models can be used to analyze the model for wave-in-ice described in the Appendix B and C. However, numerical experiment can also involve experiments using wave models such as WAM: also state-of-the-art numerical models where waves (and floating objects) can be describes by direct numerical simulations (DNS) may be a valuable research tool.

met.no do not have the sufficient expertise to carry out all these research activities alone. However, it should be pointed out that the necessary expertise to some extent exist collectively in Oslo. While met.no has strong expertise in theoretical wave model, modeling using DNS to describe wave motion (Broström, Christensen, Weber, work in progress), the oceanographic department at Oslo University has a long tradition on theoretical wave problems, including wave-in-ice studies (Prof. Jan Erik Weber and Dr. Kai Christensen). The Institute of Mechanics at Oslo University has state-of-the-art equipment for laboratory wave studies (Dr. Atle Jensen). Here wave amplitude measurements (which are needed for wave attenuation experiments) is somewhat of a local specialty. Field studies may also be required, but there is no such expertise in Oslo to the best of the authors knowledge. External groups have to be approached for such studies.

Acknowledgement

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Appendix A Some important relations on wave dynamics

There are some important relations that are used frequently in the description of waves. For convenience some of these relations are summarized below.

Let us consider a wave such that the surface elevation is given by

$$\eta(x,t) = a\sin(kx - \omega t), \qquad \text{(A 1)}$$

where a is the wave amplitude, k is the wave number and ω is its angular frequency. The energy of the surface wave, E, is given by

$$E = \frac{\rho_w g a^2}{2}, \qquad \text{(A 2)}$$

where g is gravity and ρ_w is the density of water.

The phase speed, c_p, is given by

$$c_p = \frac{\omega}{k}, \qquad \text{(A 3)}$$

while the group velocity, c_g, is given by

$$c_g = \frac{\partial \omega}{\partial k}. \qquad \text{(A 4)}$$

The group velocity is particularly important for estimating the wave amplitude as the wave energy travels with the group velocity. In a friction free case with no wave scattering processes we have for the energy flux F_E

$$F_E = c_g E = const. \qquad \text{(A 5)}$$

Accordingly, if c_g changes, for instance due to changes in water depth or the thickness of ice cover, the energy of the wave must accordingly change to keep the relation (A 5).

A note of some interest is that the flux of moment is given by

$$F_M = \frac{F_E}{c_p}. \qquad \text{(A 6)}$$

Accordingly, if the phase speed of the wave changes the momentum flux of the waves will change, provided that the energy flux is constant (as is predicted by A 5). Momentum is a conserved quantity and the quick calculation indicates that there must be some exchange of momentum with the mean flow. A more through analysis shows that the pressure fields must also be included when describing the wave-mean flow interaction but the described scenario is one component of the wave-mean flow interaction [*Broström, et al.*, 2008; *Longuet-Higgins and Stewart*, 1960; 1964].

Appendix B Waves scattered by ice floes

The main purpose of this section is to outline the “standard” theory used to describe how waves behave under the influence of an ice cover. The theory for open water is also described and the physical mechanism for the scattering of waves is described.

Let us consider a situation where open water waves enter from left (i.e., from x=-∞) and that they encounter an ice floe extending from *x*=0 to *x*=*L* having thickness *h*. The wave will propagate under the ice cover: however, as the wave speed is different under the ice cover there will be a certain reflection of the wave energy due to the ice cover, and only a certain part of the wave energy will propagate through the ice cover over to the open water at the other side of the ice floe. An important assumption in this theory is that there are non floe-floe interactions. A schematic picture of the situation is outlined in Fig. (B 1).

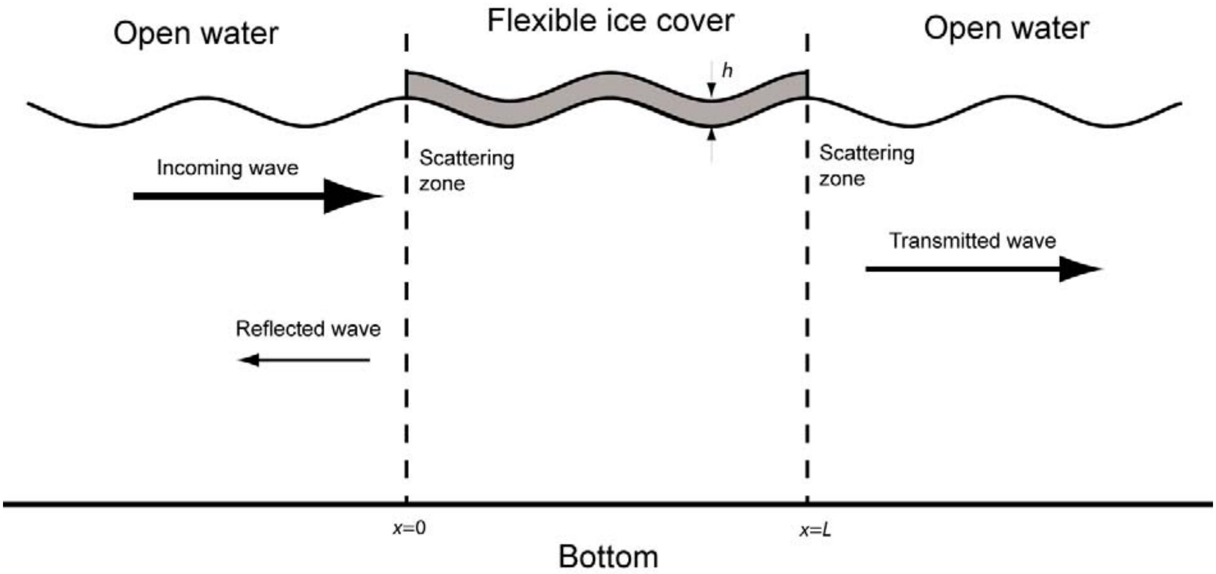

Figure B 1 Schematic picture of the physical situation when a wave encounter an ice floe.

It may be noted that the wave solution must be continuous over the entire region and the problem thus involve two different matching areas in the outlined problem. This, together with rather complicated boundary conditions at the surface, implies that the mathematical treatment of the described physical situation is not straightforward. On the contrary, the mathematical methods used for deriving the wave solutions are rather complicated and often some type on numerical optimization for matching constants must be applied. The mathematical difficulty of obtaining solutions to the described physical situation represent one of the greatest challenges for obtaining a unifying theory for waves propagating under water and sea ice. Anyway, the final product of the theory will be to predict the amplitude of the reflected wave due to the scattering from an ice floe.

B 1 Outline of equations used

B 1.1 Basic equations for the flow

Let us consider a wave such that the surface elevation is given by

$$\eta(x,t) = a\sin(kx - \omega t) \qquad \text{(B 1)}$$

where a is the wave amplitude, k is the wave number and ω is its angular frequency. For wave traveling in the x-direction the following approximate equations describe the velocity field and the pressure (here we neglect friction for simplicity, the frictional case describing waves in slush ice is outlined in Appendix C)

$$\begin{aligned} \frac{\partial u}{\partial t} &= -\frac{1}{\rho_w}\frac{\partial p}{\partial x}, \\ \frac{\partial w}{\partial t} &= -\frac{1}{\rho_w}\frac{\partial p}{\partial z} - g, \\ \frac{\partial u}{\partial x} &+ \frac{\partial w}{\partial z} = 0, \end{aligned} \qquad \text{(B 2a-c)}$$

where (u, w) are the velocities in the (x, z) direction, respectively. ρ_w is the density of water, p is pressure and g is the acceleration of gravity. It is convenient to describe the velocities by the velocity potential $\phi(x,z,t)$ such that

$$\mathbf{u} = \nabla\phi \quad \left(\text{i.e., } u = \frac{\partial \phi}{\partial x}, w = \frac{\partial \phi}{\partial z}\right). \qquad \text{(B 3)}$$

Inserting this potential into the continuity equation (B 2c) provides the following equation for the velocity potential

$$\nabla^2\phi = 0\,. \qquad \text{(B 4)}$$

This equation is often used to describe friction free wave motions: thus, boundary conditions are therefore generally derived for ϕ rather than for velocities and pressure.

B 1.2 Surface boundary conditions

The equations have to be supplemented with the dynamics of the boundaries. Notably, the boundary conditions at the surface are not straightforward and represent the main difficulty in developing theory for surface waves, and the exact form of the boundary conditions may depend upon the application. Furthermore, some simplifications must be considered and these simplifications are not always straightforward. However, here we will apply the lowest order theory. For waves there is always a kinematic condition reflecting the movement of a surface, and a dynamic conditions reflecting the physical situation we aim to describe.

B 1.2.1Kinematic condition

The kinematic condition describes the movement of a layer in the fluid (or the surface movements) and continuity of this layer implies that certain relations must hold true. The movement of a continuous layer, e.g., the free surface, is described by

$$w = \frac{d\eta}{dt} = \frac{\partial \eta}{\partial t} + u_h \cdot \nabla_h \eta\,, \qquad \text{(B 5)}$$

where index h reflects horizontal (i.e., x and y) components. Applying the definition of the velocity potential (B 3) we find

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t} + (\nabla_h \phi)\cdot(\nabla_h \eta). \qquad \text{(B 6)}$$

In general the non-linear term is considered to be small (this assumption requires that the wave amplitude is small as compared to the wave length, i.e., $a \ll \lambda$) such that

$$\frac{\partial \phi}{\partial z} = \frac{\partial \eta}{\partial t}, \qquad \text{(B 7)}$$

which will be used in this study. This boundary condition should be applied at $z=\eta$; however, by using a Taylor expansion we see that this condition can be applied at the z=0 within the order of approximation [*Phillips*, 1977].

B 1.2.2 Dynamic surface boundary condition

The dynamical boundary condition reflects the physical situation we intend to describe. Accordingly we need to use different dynamical conditions at the free water surface and at the water-ice interface. It should be noted that we will apply the surface conditions for the free surface and the ice covered surface at z=0, even though the ice may be too thick for this assumption to hold true.

Free surface

One assumption we can apply is that the pressure is zero at the sea surface: to phrase this condition in terms of the velocity potential we need to use the Bernoulli equation. We consider the Bernoulli equation to be well known and it reads

$$\frac{p}{\rho_w} + g\eta + \tfrac{1}{2}v^2 = const\,. \qquad \text{(B 8)}$$

By also including the time evolution it reads [*Phillips*, 1977]

$$\frac{\partial \phi}{\partial t} + \frac{p}{\rho_w} + g\eta + \tfrac{1}{2}v^2 = const\,. \qquad \text{(B 9)}$$

Using the velocity potential (B 3), it follows that the free surface is described by

$$\left.\frac{p}{\rho_w} + g\eta + \frac{\partial \phi}{\partial t} + \tfrac{1}{2}(\nabla \phi)^2\right|_{z=\eta} = 0\,. \qquad \text{(B 10)}$$

Thus, applying this condition at z=0 and neglecting non-linear terms we find

$$\left.\frac{p}{\rho_w} + g\eta + \frac{\partial \phi}{\partial t}\right|_{z=0} = 0 \qquad \text{(B 11)}$$

Ice conditions

Let us consider an ice floe with thickness, h. Assuming that the ice can be regarded as an elastic plate we can write a linearized relationship between the surface deflection and water pressure immediately below the ice. It should also be noted that the mass of the ice will also give a force on the water at the wave-ice interface. Following [*Liu and Mollo-Christensen*, 1988; *Wadhams, et al.*, 2002] we write

$$P_a(x,t)_{z=\eta} = -\rho_w\left(\frac{\partial \phi}{\partial t} + g\eta\right) \qquad \text{(Bernoulli equation; see B 11).}$$

$$\left.P_a(x,t)_{z=\eta} - L\frac{\partial^4 \eta(x,t)}{\partial x^4} - hP\frac{\partial^2 \eta(x,t)}{\partial x2} = \rho_i h\frac{\partial^2 \eta(x,t)}{\partial t^2}\right|_{z=0} \qquad \text{(equation of motion)} \quad \text{(B 12)}$$

The first term in (B 12) represent the pressure at the ice-water interface, the second term describes the flexural force arising from bending the elastic ice cover, the third term describes the compressive stress in the sea ice (this term is often neglected such that the assumption of a thin Bernoulli-Euler elastic plate is made), and the last term is the force required to move the mass of the ice cover. Constants in the above equation are:

$L = \dfrac{Eh^3}{12(1-s^2)}$: flexural rigidity of ice.

E: Young's modulus of elasticity, which is E=6·10^9 N m^{-2} for ice.

s: Poisson's ratio, which is s=0.3 for ice.

P: compressive stress in the ice.

Combining (B 11) and (B 12)

$$\left(L\frac{\partial^4}{\partial x^4} + hP\frac{\partial^2}{\partial x^2} + \rho_i h\frac{\partial^2}{\partial t^2}\right)\eta(x,t) = P_a(x,t)_{z=\eta} = -\rho_w\left(\frac{\partial \phi}{\partial t} + g\eta\right),$$

And we find the following dynamical condition to be fulfilled at the surface

$$\left(L\frac{\partial^4}{\partial x^4} + hP\frac{\partial^2}{\partial x^2} + \rho_i h\frac{\partial^2}{\partial t^2}\right)\eta(x,t) = -\rho_w\left(\frac{\partial \phi}{\partial t} + g\eta\right). \quad \text{(B 13)}$$

Neglecting the compressive stress in the ice we have

$$\left(L\frac{\partial^4}{\partial x^4} + \rho_i h\frac{\partial^2}{\partial t^2}\right)\eta(x,t) = -\rho_w\left(\frac{\partial \phi}{\partial t} + g\eta\right). \quad \text{(B 14)}$$

Again this condition should be applied at z=η, but will be applied at z=0, consistent within the order of approximations used in this study.

B 1.2.3 Boundary conditions at the side of the ice cover

In addition to the above conditions we must describe the dynamics of the situation at the sides of the ice cover, here denoted by x_i (note that for the situation outlined in Fig. B1, these condition should be applied at x=0 and at x=L). At the side of the ice floe, the bending moment and the shear must vanish such that

$$\left.\frac{\partial^2 \eta}{\partial x^2}\right|_{\substack{x=x_i \\ z=0}} = 0,$$
$$\left.\frac{\partial^3 \eta}{\partial x^3}\right|_{\substack{x=x_i \\ z=0}} = 0. \quad \text{(B 15a, b)}$$

In addition there must be a continuity in the velocity potential such that

$$\left.\phi^+\right|_{x=x_i} = \left.\phi^-\right|_{x=x_i},$$
$$\left.\frac{\partial \phi^+}{\partial x}\right|_{x=x_i} = \left.\frac{\partial \phi^-}{\partial x}\right|_{x=x_i}, \quad \text{(B 16a, b)}$$

where ϕ^+, ϕ^- reflects the two solutions at each side of the ice floe boundary x_i.

B 1.3 Bottom boundary condition

For many applications it is convenient to assume that the ocean is infinitely deep: In this case it is assumed that all fields are bounded at infinity. For finite depth the conditions is that the velocities is zero at the bottom.

B 1.3.1 Kinematic condition

The kinematic condition at the bottom is

$$w\big|_{z=-H} = 0, \tag{B 17}$$

implying that

$$\left.\frac{\partial \phi}{\partial z}\right|_{z=-H} = 0. \tag{B 18}$$

It should be noted that is condition is only used for the case of finite depth.

B 1.3.2 Dynamic condition

The dynamical condition at the bottom is not applicable when considering the friction free motion (the equations can only fulfill one condition at the bottom). Continuing, it should be noted that friction free conditions imply that the solution will unphysical predict that there is a certain horizontal velocity at the bottom. In a real fluid the velocity will be zero at a solid surface, by including friction this condition will be fulfilled by a thin frictional layer.

B 1.4 Dispersion relation

The dispersion relation describes the relation between the wave period and the wave length. Furthermore, since the phase speed, c_p, is given by

$$c_p = \frac{\omega}{k},$$

and the group velocity, c_g, is given by

$$c_g = \frac{\partial \omega}{\partial k}.$$

these important quantities can be calculated directly from the dispersion relation. Another important aspect of the dispersion relation with respect to waves and ice is that the dispersion relation will depend both on the wave length and the ice thickness, h, i.e.

$$\omega(k,h) = 0. \tag{B 19}$$

Thus, if we know ω and k from measurements we can calculate the thickness h. There have been some attempts to estimate the wave properties by SAR methods and to use (B 19) to calculate the ice thickness; the procedure is not entirely straightforward and the results are ambiguous [*Nagurny, et al.*, 1994; *Wadhams, et al.*, 2002]. The problem is most likely that the relation (B 34) is not known to the required degree.

To solve the equations (2.7)–(2.11) one makes an *ansatz*, or insightful guess, of how the solution of the equations looks like. Using this ansatz we can derive certain relations that must hold for the ansatz to be a proper solution of the boundary conditions. Let us assume that the solution takes the form

$$\phi = \sum_n \left(A_n \exp(ik_n x) + B_n \exp(-ik_n x)\right)\exp(-k_n z)\exp(i\omega t)\,; \qquad \text{(B 20)}$$

this equation fulfils the boundary condition if, and only if, ω and k are related as

$$gk_n - \omega^2 = 0 \qquad \text{(open water)} \qquad \text{(B 21a)}$$

$$Lk_n^5 - \left(\rho_w g - \rho_i h\omega^2\right)k_n - \rho_w \omega^2 = 0 \qquad \text{(ice condition)} \qquad \text{(B 21b)}$$

which is know as the dispersion relation for open water and ice, or flexural gravity, conditions, respectively. (Here note that h can be viewed as a equivalent ice thickness for ice concentration times the thickness of the slushy layer such that $h = \left(c_p h_p + c_f h_f\right)$, where index p reflect pancake ice and index f frazil ice, respectively)

Let us first consider the simplified case with L=0: the dispersion relations become

$$k_n = \frac{\omega^2}{g} \qquad \text{(open water)} \qquad \text{(B 22a)}$$

$$k_n = \frac{\omega^2}{g - \rho_i h\omega^2} \qquad \text{(ice condition with added mass, i.e., pancake ice)} \qquad \text{(B 22b)}$$

It should be noted that for the case with small frequencies, i.e., long waves, such that $g >> \rho_i h\omega^2$ the dispersion relation is similar to that of open water. Equation (B 22b) shows that the ice thickness can be estimated if the wave frequency and the wave length are known. However, this relation do not consider the stiffness of the ice (flexural force), or the fact that the ice conditions often are characterized by presence of very viscous slush ice.

For $L\neq 0$ it is not possible to find a simple relationship between k and ω. However there are some simple limits that are useful. It can be shown [*Fox and Haskell*, 2001] that the behavior of the solution changes character at $\omega_c = \left(\rho g^5/L\right)^{1/8}$: For $\omega < \omega_c$, $k \approx \omega^2/g$ while for $\omega > \omega_c$ $k \approx \left(\rho\,\omega^2/L\right)^{1/5}$. For the first case the dispersion relation equals the relation of the open water.

The case with $L\neq 0$ furthermore implies that the group velocity of the waves can become very small for certain values in the ice stiffness. If this happens, the wave energy will be trapped in this region and if waves continue to pump energy into such region the wave amplitude will ultimately become very large. This has been one of the explanation for severe wave condition appearing deep inside the Antarctic pack ice during an expedition with RV Polarstern in 1986 [*Liu and Mollo-Christensen*, 1988].

Appendix C Frictional affects

The presence of strong wave motions implies that small ice particles (order mm) cannot attach to each other such that larger pieces of ice cannot be formed. Under freezing conditions with intense wave action very viscous and buoyant ice slush is created. Apparently, the ice "cover" in this case do not have any flexural force on the wave action such the model in Appendix B cannot be used. To describe this situation, a model based on a two layer system can be outlined to describe the most pertain parts of the slush ice system [*De Carolis and Desiderio*, 2002; *Weber*, 1987]. The upper layer consists of viscous and buoyant slush ice while the lower layer consists of water. A schematic picture of the physical scenario described in the Appendix is presented in Fig. (C 1). This type of system was initially described by Weber (1987) using a Lagrangian approach; however, as this mathematical framework is not common to most geophysical fluid scientists we outline a similar model based on the more standard Eulerian framework in this Appendix[10].

Another physical situation of great interest is the pancake ice field characterized by many small (order 0.1-1m but with cakes up to 10 m size) ice floes surrounded by slush ice. This type of ice is most likely very difficult to model in detail. The question is what type of model that should be applied to describe waves in this important region. Observations on the dispersion relation in pancake ice areas indicate that the model described in this Appendix describes data relatively well, albeit the problem in determining the viscosity for the pancake ice layer [*Wadhams, et al.*, 2004]. Another situation of importance is the MIZ characterized by very small ice floes and with a high density of ice pieces that can be described as "slush ice" (notably, slush ice is used for a water-ice mixture of very small ice crystals; water-ice mixture with larger ice pieces of order cm to m appears not to have any specific name to the authors knowledge).

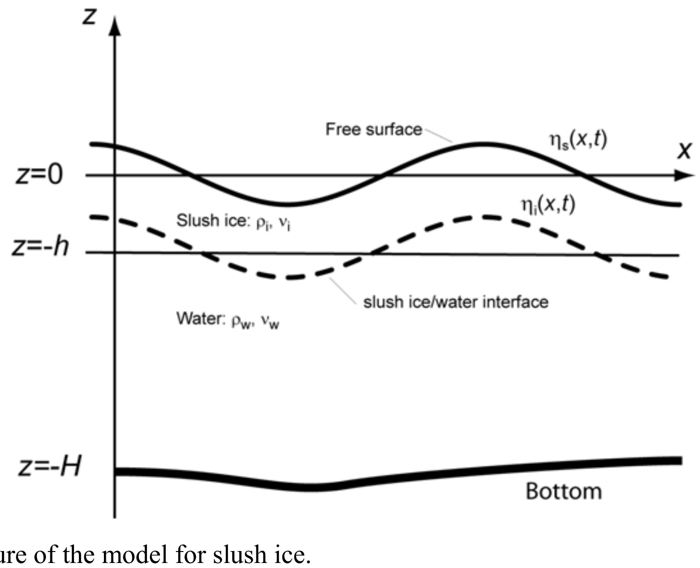

Fig C 1 Schematic picture of the model for slush ice.

[10] The mathematical treatment of the waves can be outlined in Eulerian framework (most common) or the Lagrangian framework. In this study we use the Eulerian framework although it may be noted that many properties of waves are more naturally described using a Lagrangian framework (Lamb, H. 1932; Weber, .1987)

C 1 Equations for the flow

Describing the frictional affect implies that we need to consider the frictional terms in the Navier-Stokes equations: However, we again assume that we deal with small amplitude waves and neglect the non-linear terms; thus,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial x^2},$$
$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\frac{\partial^2 w}{\partial x^2} - g, \qquad \text{(C 1a-c)}$$
$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0,$$

As in Appendix B we will use the velocity potential $\phi(x,z,t)$: However, the inclusion of friction will create frictional boundary layers that carries some vorticity, and its is it is necessary to also include a rotational component of the flow, $\mathbf{u}_r$, which is related to the streamfunction, $\psi(x,z,t)$. Let us consider the following definitions [*Lamb*, 1932]

$$\mathbf{u}_\phi = \nabla\phi,$$
$$\mathbf{u}_\psi = \nabla\times\psi. \qquad \text{(C 2a, b)}$$

The velocity field is thus given by

$$\mathbf{u}_{tot} = \mathbf{u}_\phi + \mathbf{u}_\psi = \nabla\phi + \nabla\times\psi\ , \qquad \text{(C 3)}$$

i.e., $u = \frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial y}, w = \frac{\partial\phi}{\partial y} + \frac{\partial\psi}{\partial x}$. Using these definitions the linear Navier-Stokes equations can be expressed as

$$\nabla\left(\frac{\partial\phi}{\partial t} + \frac{P}{\rho} - \varphi\right) = \nabla\times\left(\frac{\partial\boldsymbol{\psi}}{\partial t} + \upsilon\nabla^2\boldsymbol{\psi}\right), \qquad \text{(C 4)}$$

where φ is the potential arising from the gravity. We thus have [*Lamb*, 1932]

$$\nabla^2\phi = 0$$
$$\frac{\partial\phi}{\partial t} + \frac{P}{\rho} - \varphi = 0 \qquad \text{(C 5a-c)}$$
$$\frac{\partial\boldsymbol{\psi}}{\partial t} + \upsilon\nabla^2\boldsymbol{\psi} = 0$$

C 2 Surface boundary conditions

C 2.1 Kinematic condition

Following [*Lamb*, 1932] we write the kinematics condition (here assuming that the boundary condition can be evaluated at z=0 rather than at z=η within the order of approximation applied in this study; this approximation can be derived by considering a Taylor expansion of all variables around z=0 and neglecting all terms that includes the small amplitude a):

$$w = \frac{d\eta}{dt} = \frac{\partial\eta}{\partial t} + u_h\cdot\nabla_h\eta\ , \qquad \text{(C 6)}$$

where index h reflects horizontal (i.e., x and y) components. Applying the definition (C 3) and assuming that the non-linear term is small we find

$$\left(\frac{\partial \phi}{\partial z}+\frac{\partial \psi}{\partial x}\right)\Bigg|_{z=0}=\frac{\partial \eta}{\partial t} \qquad \text{(C 7)}$$

C 2.2 Dynamical condition

The dynamical boundary conditions are zero stress at the surface

$$\begin{aligned} &\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\rho \nu = 0, \quad x\text{-dir;} \\ &-P + 2\rho\nu\frac{\partial w}{\partial z} = 0, \quad z\text{-dir;} \end{aligned} \qquad \text{(C 8a, b)}$$

Using (C 5c) we can also write (C 8b) as

$$\frac{\partial \phi}{\partial t} - g\eta + 2\nu\frac{\partial w}{\partial z} = 0\,,$$

writing the u, w-velocities as a velocity potential we thus get the surface boundary conditions

$$\begin{aligned} &2\frac{\partial^2 \phi}{\partial x \partial z}+\frac{\partial^2 \psi}{\partial x^2}-\frac{\partial^2 \psi}{\partial z^2}=0, \qquad x\text{-dir;} \\ &\frac{\partial \phi}{\partial t} - g\eta + 2\nu\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}+\frac{\partial \psi}{\partial x}\right)=0, \quad z\text{-dir;} \end{aligned} \qquad \text{(C 9a, b)}$$

C 3 Bottom boundary conditions

There are two types of bottom boundary conditions that are frequently used: Either of the following are used

- A requirement that the solution is bounded when extending toward infinity or
- zero velocity at the bottom (here recalling that we consider frictional affects): Thus

$$\begin{aligned} u &= 0, \\ w &= 0, \end{aligned} \qquad \text{(C 10a, b)}$$

or expressed in terms of the potentials

$$\begin{aligned} &\left(\frac{\partial \phi}{\partial z}-\frac{\partial \psi}{\partial x}\right)\Bigg|_{z=-H}=0, \\ &\left(\frac{\partial \phi}{\partial z}+\frac{\partial \psi}{\partial x}\right)\Bigg|_{z=-H}=0. \end{aligned} \qquad \text{(C 11a, b)}$$

C 4 Interfacial boundary conditions

If we assume that there is more than one layer (in this section we consider an upper buoyant high-viscosity slush ice layer and a lower layer consisting of ordinary sea water) the following must hold at the interface between the layers: The horizontal and vertical velocities must be continuous

$$\left(\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^+}=\left(\frac{\partial\phi}{\partial z}-\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^-},$$
$$\left(\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^+}=\frac{\partial\eta}{\partial t}\bigg|_{z=z_i}=\left(\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^-}. \qquad \text{(C 12a, b)}$$

There must also be continuity in the stresses (see C 9)

$$2\frac{\partial^2\phi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\bigg|_{z=z_i^-}=2\frac{\partial^2\phi}{\partial x\partial z}+\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial z^2}\bigg|_{z=z_i^+}, \qquad x\text{-dir;}$$
$$\frac{\partial\phi}{\partial t}-gz_i^-+2\nu\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^-}=\frac{\partial\phi}{\partial t}-gz_i^++2\nu\frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial z}+\frac{\partial\psi}{\partial x}\right)\bigg|_{z=z_i^+}, \quad z\text{-dir.}$$

(C 13a, b)

Here the superscripts (+) and (-) refers to the layer above and below the interface, respectively.

C 5 Solutions

The solutions can be written as [*De Carolis and Desiderio*, 2002; *Lamb*, 1932]

$$\phi=\left(Ae^{kz}+Be^{-kz}\right)e^{i(kx-\omega t)},$$
$$\psi=\left(Ce^{\alpha z}+De^{-\alpha z}\right)e^{i(kx-\omega t)}, \qquad \text{(C 14a, b)}$$

where (applying C 5c)

$$\alpha^2=k^2-i\frac{\omega}{\upsilon}. \qquad \text{(C 15)}$$

If we assume that the basin is infinitely deep it follows that A=C=0 to fulfill the condition of non-infinite numbers at infinity, and we have

$$\phi=Be^{-kz}e^{i(kx-\omega t)},$$
$$\psi=De^{-\alpha z}e^{i(kx-\omega t)}.$$

However, for finite depth systems the full set of equations must be considered.

If a single layer system is considered Eqs. (C 14) contains 4 unknowns. Now, the kinematic condition at the surface (C 7) can be used to relate η to ϕ and ψ; two boundary conditions at the surface (C 9) and two boundary conditions at the bottom (C 11) implies that the 4 unknowns can be determined using four equations and the problem is well posed. For the case with two layers (as is the case when describing slush ice on top of pure water), Eqs (C 14) must be applied for each layer and there will be four new constants to determine (totally eight constants): these four new constants can be determined by using the four conditions at the interface (C 12a, b) and (C 13a, b). However, is not a straightforward task to determine the eight constant analytically from the eight conditions, and numerical methods to find the constants must be used [*De Carolis and Desiderio*, 2002]. We have not solved the system of equation as a part of this literature review and rely mainly on other studies for exemplifying the solutions [*De Carolis and Desiderio*, 2002]. Some results from the model are presented in Figs (C2, C3). For more discussion see Sec. 2.3.3.

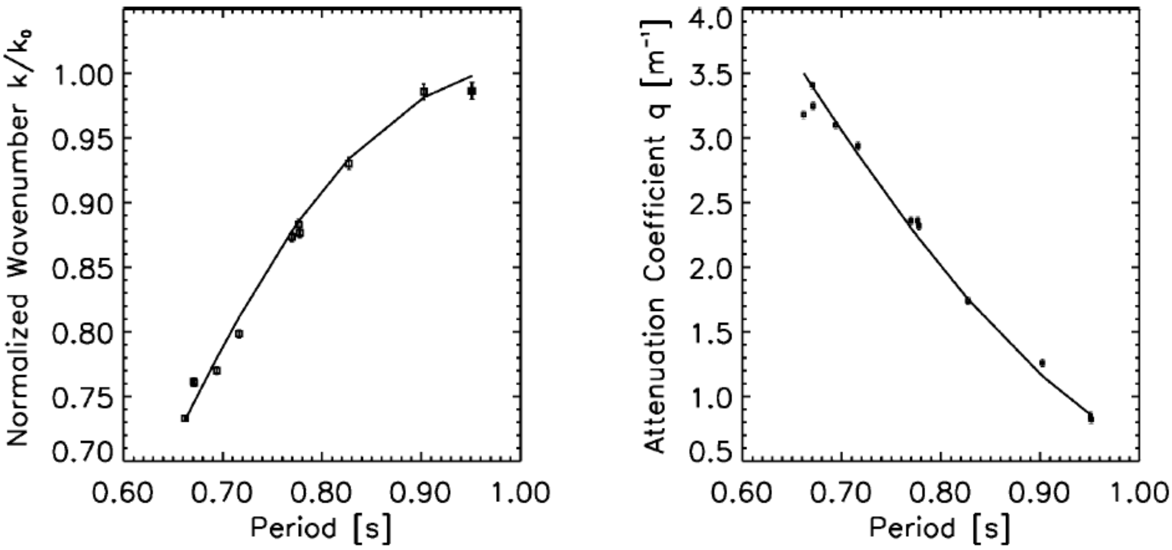

Figure C 2 Comparison between model results and laboratory experiment carried out with waves in slush ice [*Newyear and Martin*, 1997]. Left figure shows the dispersion relation and the right figure displays the attenuation coefficient. The viscosity of the upper layer was used to fit data and left figure uses ν_i=2.94 10^{-2} m^2 s^{-1} while the right panel uses ν_i=3.68 10^{-2} m^2 s^{-1}, in lower layer ν_i=1.8 10^{-6} m^2 s^{-1} were used.

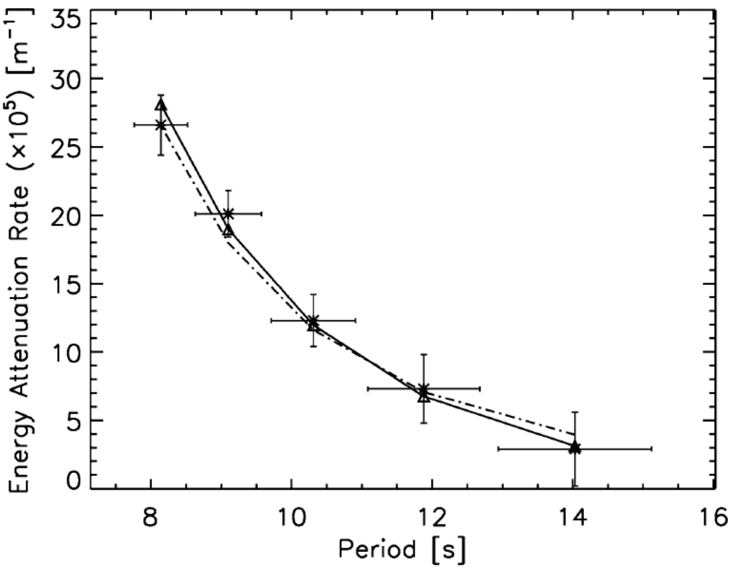

Figure C 3 Comparison of data from Greenland 4 September 1979 [*Wadhams, et al.*, 1988] and curves with the Weber model (dash dot line) and the viscous two-layer model presented here. The viscosity was used to fit model with data and ν_i=1.95 10^{-3} m^2 s^{-1} was used for the present model while Weber used ν_i=1.95 10^{-3} m^2 s^{-1}.

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